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Money runs[☆]

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ABSTRACT

In this paper, we develop a model that links the funding role of bank debt in the primary market to its circulation in the secondary market. We uncover a new rationale for why banks do what they do. Banks choose to fund themselves with demandable debt because it commands a high price, even though doing so exposes them to "money runs" resulting from its failure to circulate. In the model, banks endogenously perform the essential functions of real-world banks: they transform liquidity, transform maturity, pool assets, and have dispersed depositors. We show novel effects of suspension of convertibility.

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1. Introduction

What is a bank? For one thing, a bank is something that creates liquidity by issuing circulating liabilities. These include not only the deposit you now exchange electronically by debit card or bank transfer,[1] but also instruments exchanged by

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hand historically, such as bank-drawn bills of exchange in early modern Europe, bank-issued notes in the 19th-century US, and bank-certified checks more recently.

Before it circulates in the secondary market, bank debt is issued to raise funds in the primary market. Thus, when banks choose what security to issue, they should take its secondary-market liquidity into account. In practice, banks commonly choose to issue securities, like banknotes and deposits, that are redeemable on demand.

But such demandable debt can be illiquid. Indeed, many bank panics and financial crises throughout history, from 18th-century London to contemporary Greece, seem to have followed from the failure of bank debt to circulate (see Section A.2). In such crises, convertibility is often suspended. This mechanically prevents bank runs—you cannot run a bank if you cannot redeem on demand. But it has been argued that it could also impede circulation—you could be unlikely to accept the debt as payment if you cannot redeem it on demand.[2] Remarkably, however, this has not always been the case. To the contrary, bank debt sometimes resumes circulation when convertibility is suspended.[3]

Despite historical precedents, most current theories of why banks choose to issue fragile financial securities are not linked to their secondary-market circulation (e.g., Calomiris and Kahn, 1991; Diamond and Dybvig, 1983, and Diamond and Rajan, 2001b).[4] To develop a theory based on this link, we model how bank debt circulates in the secondary market explicitly, following the search-and-matching literature (see, e.g., Lagos et al., 2017 for a survey).

We use the model to address the following questions. Why is bank debt so often redeemable on demand, regardless of the form it takes, from physical banknotes to electronic deposits? Why is demandable debt subject to liquidity crises? Given it is, why do banks still choose to issue it, exposing themselves to sudden redemptions, and making the financial system fragile too? And what should financial regulators do about it? In particular, does suspending convertibility necessarily come with the cost of impeding circulation?

By linking the financing role of bank debt in the primary market to its circulation in the secondary market, we uncover a new rationale for why banks do what they do. Banks choose to fund themselves with demandable debt to take advantage of a “price effect of demandability”: demandable debt trades at a high price in the secondary market, and hence decreases banks’ cost of funding in the primary market. But this high price is not always a good thing. Reluctant to pay it, potential counterparties may decide not to buy the debt at all, and therefore leave whoever holds it with something he cannot trade, but can only redeem on demand. Such redemption constitutes a new kind of bank run, a “money run,” resulting entirely from the failure of debt to circulate in the secondary market. However, banks continue to issue demandable debt. To do so, they exploit economies of scale that arise solely from the price effect of demandability. Specifically, they transform liquidity, transform maturity, pool assets, and borrow from dispersed depositors. I.e. they do something that looks like real-world banking. But, to do it effectively, they exacerbate their exposure to money runs. Suspension of convertibility can not only prevent runs in a crisis, but can, in fact, facilitate circulation.

Model preview. Because we want to show how banking can arise endogenously, we start with a single borrower B with a single investment. Ultimately, multiple borrowers will form an institution that assumes features of real-world banks. But, for now, B resembles a bank only insofar as its debt plays a dual role. To capture its role in raising funds, we assume that B is penniless and needs to fund an investment from a creditor C_0 (i.e. a depositor). To capture its role in providing liquidity, we make two assumptions. First, C_0 could be hit by a liquidity shock before B’s investment pays off, as in Diamond and Dybvig (1983). Thus, C_0 could want to trade B’s debt to get liquidity. Second, C_0 must trade bilaterally in a decentralized market, similar to those in Trejos and Wright (1995) and Duffie, Garleanu and Pedersen (2005). We assume that to acquire B’s debt from C_0, a counterparty C_1 must pay an entry cost k to enter and bargain with C_0 over the price. Likewise, if C_1 is shocked, a counterparty C_2 must pay k and bargain with him to trade, and so on. The terms of trade between counterparties depend on how B designs its debt. In particular, B can make its debt redeemable on demand. In this case, B chooses a redemption value r, for which a creditor can redeem before the investment pays off. To pay r, B has to liquidate its investment (so r cannot be greater then B’s liquidation value).

Results preview. Our first two main results capture a trade-off of increasing the redemption value r. First, a high redemption value has a bright side: it allows B to borrow more and more cheaply as C_0 is willing to pay more for demandable debt, even if he never redeems in equilibrium. The reason is that C_0 values the option to redeem off equilibrium, even if he never exercises it, because it provides him with a valuable threat (i.e. outside option) when he bargains with C_1, a threat that is more valuable if he can redeem for more, i.e. as r increases. As a result, he can sell B’s debt to C_1 at a higher price. Anticipating selling at a higher price in the secondary market, he is willing to lend more to B in the primary market. This result contrasts with existing models of demandable debt as liquidity insurance, in which, roughly, you do not need the option

[2] See, e.g., Dewald (1972) on how the “trade journals reported that depression was accountable to suspension and a lack of loans to sustain trade” (p. 939), i.e. on how some argued that the suspension impeded payments/trade. See also Sprague (1910).

[3] Take the crisis of 1907, for example, in which commercial bank clearing houses acted, to an extent, like central banks, inter alia, issuing receipts that served as a means of payment (Gorton and Mullineaux, 1987). Convertibility was suspended; the debt could not be redeemed on demand. But, nonetheless,

to redeem debt on demand if you can just trade it in the secondary market (e.g., Jacklin, 1987). Here, in contrast, you do: just the option to redeem on demand props up the resale price of debt in the secondary market, even if the option is never exercised. We refer to this as the "price effect of demandability," because it works entirely through the secondary market price, not through actual redemptions.

Second, in contrast, a high redemption value has a dark side. Although it increases the price C_0 can sell for, it symmetrically increases the price C_1 must pay. This makes C_1 less willing to enter. Thus, for high r, C_0's option to redeem on demand can undermine itself, putting him in such a strong bargaining position that he has no willing counterparty, and ends up redeeming on demand.

Our third main result is an equilibrium characterization. We show conditions under which B chooses to borrow via demandable debt and sets the highest possible redemption value, even though doing so makes him susceptible to a new kind of run. Like fiat money, B's debt could stop circulating due to a sudden (but rational) change in beliefs. But, unlike fiat money, B's debt can be redeemed on demand in a bank run. Hence, the fragility of B's debt in the secondary market leads to the fragility of B itself. In contrast to the literature following Diamond and Dybvig (1983), such a run can occur even though B has only a single creditor—there is not a coordination problem in which multiple creditors race to withdraw; rather, there is a coordination problem in which a creditor cannot get liquidity in the secondary market and must withdraw as a result. We refer to this run as a "money run," given it is the result of the failure of B's debt to function as money in the secondary market.

For our fourth main result, we turn to a policy that has been used to prevent runs historically: suspension of convertibility. We show that it not only mechanically puts an end to bank runs in a crisis, as in Diamond and Dybvig (1983), but it can also potentially restore the circulation of bank debt and, thereby, even increase B's borrowing capacity. The reason is that it lowers its price, and thus makes counterparties more willing to enter. We find, however, that a suspension policy must be sufficiently aggressive to be effective. A mild policy might not lower the price enough to restore circulation, in which case it does nothing but take away debtholders' redemption option.

Our fifth main result is that if multiple borrowers can get together, they can exploit economies of scale that allow them to issue debt with total redemption value in excess of the total liquidation value of their investments. To show this, we consider N parallel versions of the model—we assume that there are N parallel borrowers, each of which borrows to fund an investment from one of N parallel creditors, each of whom trades bilaterally in one of N parallel markets. The only link between the parallel versions is that the borrowers can issue debt backed by the entire pool of investments. So now there are N creditors holding N securities backed by N investments, instead of one creditor holding one security backed by one investment. We assume that everything is perfectly correlated, so, unlike in Diamond and Dybvig (1983) and Diamond (1984), there is no possibility of diversification. Despite this, we find that getting together can still benefit borrowers, because they can give each of the N creditors the option to redeem for the entire pool.

Why does each creditor have a claim on the entire pool, rather than just a fraction $1/N$ of it? Because if bank debt circulates, no one redeems on the equilibrium path; thus, if one creditor deviates, he is the only one redeeming, and he has the first claim on all of the assets. As per the price effect of demandability, the option to be first in line is valuable, even if it is never exercised. Hence, it can be enjoyed by one creditor without making it unavailable to others—in the language of public goods, the redemption option is "non-rivalrous." To decrease their cost of funding, the borrowers continue increasing the redemption value r until the price of their debt is so high that counterparties are just indifferent between paying the entry cost k and staying out. Remarkably, by doing so, the borrowers can fund exactly the investments with positive social surplus—no more and no fewer.

With this result, we see that our model, based on only the dual role of bank debt, points to a new rationale for why banks do what they do: borrowers form a "bank" (or a banking system) only to create demandable debt; they endogenously transform liquidity, transform maturity, pool assets, and borrow from dispersed creditors. And, like a bank, they are fragile. By doing banking, borrowers exacerbate their exposure to money runs. Unlike in the banking literature, banks are fragile because circulation is fragile. And, unlike in the new monetarist literature, circulation can be fragile no matter how small counterparties' entry cost k is. The reason is that, here, the redemption value r is determined endogenously. If k decreases, the bank responds by increasing r, keeping counterparties indifferent to entry. Thus, as $k \to 0$, r approaches the face value—as in developed economies in normal times, bank debt is redeemable at par. But, given counterparties are indifferent to staying out, the debt remains a fragile means of payment. The bank remains vulnerable to money runs, which can now trigger liquidation of the whole pool of investments.

Further results. We explore three extensions. (i) We add random, heterogenous entry costs. We show that this is another way to generate runs on the equilibrium path, as well as to obtain a unique equilibrium. (ii) We show that if B can choose its investment, its choice can be distorted toward high-liquidation-value investments, which facilitate its issuing demandable debt. (iii) We study a version of the model with a continuum of creditors in which debt can be rolled over as well as traded. We show that the results of our baseline model are robust. (This setup also has the attractive feature that not every withdrawal is a run.)

Layout. The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes benchmarks

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	not demandable	demandable
non-tradeable	"loan"	"puttable loan"
tradeable	"bond"	"banknote"

Fig. 1. Debt Instruments.

2. Model

In this section, we present the model.

2.1. Players, dates, and technologies

There is a single good, which is the input of production, the output of production, and the consumption good. Time is discrete and the horizon is infinite, $t \in \{0, 1, \ldots\}$.

There are two types of players, a borrower B and infinitely many deep-pocketed creditors C_0, $C_1, \ldots$, where C_t is "born" at Date t. Everyone is risk-neutral and there is no discounting. B is penniless but has a positive-NPV investment. The investment costs c at Date 0 and pays off $y > c$ at a random time in the future, which arrives with intensity ρ. Thus, the investment has NPV $= y - c > 0$ and expected horizon $1/\rho$. B may also liquidate the investment before it pays off; the liquidation value is $\ell < c$.

B can fund its investment by borrowing from C_0. However, there is a horizon mismatch similar to that in Diamond and Dybvig (1983): creditors may need to consume before B's investment pays off. Specifically, creditors consume only if they suffer "liquidity shocks," which arrive at independent random times with intensity θ (after which they die). Hence, a creditor's expected "liquidity horizon" is $1/\theta$.

For now, we focus on a single borrower funding a single investment with debt to a single creditor; this helps us to distinguish the forces in our model from those in the literature.[5] Later, we include multiple borrowers funding multiple investments from multiple creditors; this allows us to show how the forces in our model give rise to something that looks like real-world banking.

2.2. Borrowing instruments

At Date 0, B borrows the investment cost c from its initial creditor C_0 via an instrument with terminal repayment $R \leq y$, paid when the investment pays off, and redemption value $r \leq \ell$, paid if the instrument is redeemed earlier.[6] Creditors can exchange the instrument among themselves and B must repay whichever creditor holds it. Hence, the instrument is tradeable demandable debt, and we refer to it as a "banknote." We let v_t denote the Date-t value of B's debt to a creditor not hit by a liquidity shock.

As benchmarks, we consider instruments that may not be tradeable (so B has to repay C_0) and/or may not be demandable, but may be "long-term" (so B makes only the terminal repayment). I.e. we allow B to borrow via the banknote or one of the following debt instruments: (i) non-tradeable long-term debt, which we refer to as a "loan," (ii) non-tradeable demandable debt, which we refer to as a "puttable loan"; and (iii) tradeable long-term debt, which we refer to as a "bond" (although it also resembles an equity share as debt and equity have equivalent payoffs since the terminal payoff y is deterministic). These instruments are summarized in Fig. 1.

2.3. Secondary debt market: entry, bargaining, and settlement

If B has borrowed via tradeable debt, then creditors can trade it bilaterally in a decentralized market. At each Date t, C_t is the single (potential) counterparty with whom the debtholder, denoted by H_t, can trade B's debt. Whenever C_t pays an "entry" cost k he meets H_t, in which case C_t and H_t determine the price p_t via generalized Nash bargaining. H_t's bargaining power is denoted by η. If C_t and H_t agree on a price, then trade is settled: C_t becomes the debtholder in exchange for p_t units of the good. Otherwise, H_t retains the debt. If the debt is demandable, H_t can demand redemption from B or he can remain the debtholder at Date $t+1$. This sequence of entry, bargaining, and settlement is illustrated in Fig. 2.[7]

[5] For example, there is no coordination problem among multiple creditors (but we show there can be a different coordination problem with a single creditor) and there is no possibility to pool multiple investments (but we show a new reason to pool investments in an enriched environment (Section 4.4)).

[6] Observe that, combined with the assumption $\ell < c$ above, the assumption that $r < \ell$ implies that the redemption value is less than the amount lent. In

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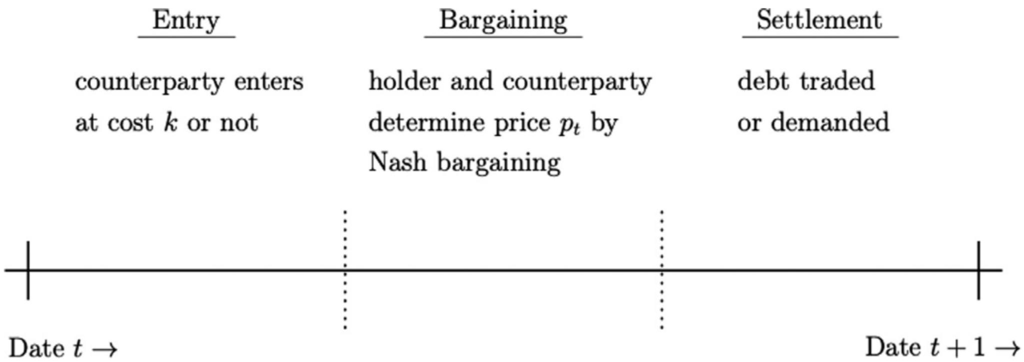

Fig. 2. Secondary-market Trade.

The settlement stage is a new modeling feature that allows us to model demandable debt. But the entry and bargaining stages are standard in the literature. The entry cost k can capture the physical costs of coming to market or any opportunity cost of doing so.[8] Bargaining, which is common not only in opaque wholesale and financial markets, but also in transparent retail markets (Shelegia and Sherman, 2019), can capture any imperfect ability to commit to posted prices (see also Section 5).

We let σ_t denote C_t's mixed strategy if H_t is hit by a liquidity shock: $\sigma_t = 1$ means that C_t enters for sure and $\sigma_t = 0$ means that C_t does not enter. Thus, σ_t also represents the probability that H_t finds a counterparty when hit by a liquidity shock.[9]

2.4. Sunspots

At each date, an exogenous state variable $s_t \in \{0, 1\}$ is realized. s_t is not payoff relevant; rather, it is a "sunspot" variable that can serve to coordinate beliefs. Later, we will interpret $s_t = 1$ as "normal times" and $s_t = 0$ as a "confidence crisis." We assume that $s_0 = 1$, that $\mathbb{P}[s_{t+1} = 0 \,|\, s_t = 1] =: \lambda$, and that $\mathbb{P}[s_{t+1} = 0 \,|\, s_t = 0] = 1$, where we think about λ as a small number. In words: the economy starts in normal times and a permanent[10] confidence crisis occurs randomly with small probability λ.

2.5. Timeline

First, B makes C_0 a take-it-or-leave-it offer of a repayment and a redemption value, as described in Section 2.2 above. Then, if C_0 accepts, he becomes the initial debtholder H_1. The debtholder may redeem on demand or may trade in the secondary market, as described in Section 2.3 above. Formally, the extensive form is as follows.

Date 0

B offers C_0 a repayment R and a redemption value r.
If C_0 accepts, then B invests c. C_0 is the initial debtholder, $H_1 = C_0$.

- *Date* $t > 0$

If B's investment pays off: B repays R to H_t and B consumes $y - R$.
If B's investment does not pay off: s_t is realized; there is entry, bargaining, and settlement as described in Section 2.3.
If there is trade, C_t becomes the new debtholder, $H_{t+1} = C_t$.
If there is no trade, H_t either holds the debt, $H_{t+1} = H_t$, or redeems on demand, in which case B liquidates its investment, repays r to H_t, and consumes $\ell - r$.

2.6. Equilibrium

The solution concept is Markov perfect equilibrium, where the sunspot s_t is the Markov state. An equilibrium constitutes (i) the repayments R and r, (ii) the price of debt in the secondary market p_t at each date, and (iii) the entry strategy σ_t of the potential counterparty C_t such that B's choice of instrument and C_t's choice to enter are sequentially rational, p_t is

[8] Note that what matters is just that C_t makes the decision to bear a cost to trade before he bargains with H_t and that C_t and H_t split the gains from

determined by Nash bargaining, and each player's beliefs are consistent with other players' strategies and the outcomes of Nash bargaining.

As s_t must be a sufficient statistic for the endogenous variables, we can replace the history-dependence, indicated by the subscripts, with state-dependence, indicated by a superscript: $v_t = v^{s_t}$, $\sigma_t = \sigma^{s_t}$, and $p_t = p^{s_t}$.

3. Benchmarks

To begin, we consider three benchmark instruments, the loan, the puttable loan, and the bond. We verify two results in the literature in our environment: (i) demandability can increase debt capacity as in Calomiris and Kahn (1991) and (ii) tradeability can substitute for demandability as in Jacklin (1987).

3.1. Loan

First, we consider a loan, i.e. non-tradeable long-term debt. At Date t, the value v^{s_t} of the loan with face value R can be written recursively:

$$v^{s_t} = \rho R + (1-\rho)(1-\theta)\mathbb{E}_t[v^{s_{t+1}}]. \tag{1}$$

The terms are determined as follows. With probability ρ, B's investment pays off and B repays R. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder H_t is hit by a liquidity shock. Since the loan is neither tradeable nor demandable, H_t gets zero. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains B's debt at Date $t+1$, which has value $\mathbb{E}_t[v^{s_{t+1}}]$ at Date t since there is no discounting. Since v^{s_t} does not depend on σ^{s_t} and s_t is payoff irrelevant, $v^{s_t} \equiv v$ and Eq. (1) gives

$$v = \frac{\rho R}{\rho + (1-\rho)\theta}. \tag{2}$$

Even though B will always repay eventually, the loan's value v is less than its face value R. The loan is discounted because, without the option to demand debt or trade it, H_t gets nothing in the event of a liquidity shock. Hence, the discount vanishes as shocks become unlikely, $v \to R$ as $\theta \to 0$. For $\theta > 0$, demandability and tradeability can help to reduce the discount, as we see next.

3.2. Puttable loan

Now we consider a puttable loan, i.e. non-tradeable demandable debt. At Date t, the value v^{s_t} of the puttable loan can be written recursively:

$$v^{s_t} = \rho R + (1-\rho)\Big(\theta r + (1-\theta)\mathbb{E}_t[v^{s_{t+1}}]\Big). \tag{3}$$

The terms are determined as follows. With probability ρ, B's investment pays off and B repays R. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder H_t is hit by a liquidity shock. Since the loan is demandable, but not tradeable, H_t redeems on demand and gets r. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains B's debt at Date $t+1$, which has value $\mathbb{E}_t[v^{s_{t+1}}]$ at Date t since there is no discounting. Since v^{s_t} does not depend on σ^{s_t} and s_t is payoff irrelevant, Eq. (3) gives

$$v = \frac{\rho R + (1-\rho)\theta r}{\rho + (1-\rho)\theta}. \tag{4}$$

We now compare the puttable loan's debt capacity with the loan's, where "debt capacity" refers to the maximum B can borrow given limited liability. I.e. we compare Eq. (4) with $R = y$ and $r = \ell$ and Eq. (2) with $R = y$:

Proposition 1. *(Benchmark: benefit of demandability.) If*

$$\frac{\rho y}{\rho + (1-\rho)\theta} < c \leq \frac{\rho y + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta}, \tag{5}$$

then B can fund itself with a puttable loan but not with a loan.

The analysis so far already points to one rationale for demandable debt. As in Calomiris and Kahn (1991), the ability to

3.3. Bond

Now we consider a bond, i.e. tradeable long-term debt. At Date t, the value v^{s_t} of the bond can be written recursively:

$$v^{s_t} = \rho R + (1-\rho)\Big(\theta\sigma^{s_t}p^{s_t} + (1-\theta)\mathbb{E}_t[v^{s_{t+1}}]\Big). \tag{6}$$

The terms are determined as follows. With probability ρ, B's investment pays off and B repays R. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder H_t is hit by a liquidity shock. Since the bond is tradeable, but not demandable, H_t gets p^{s_t} if he finds a counterparty, which happens with probability σ^{s_t}, and nothing otherwise. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains B's debt at Date $t+1$, which has value $\mathbb{E}_t[v^{s_{t+1}}]$ at Date t since there is no discounting.

To solve for the value v^{s_t}, we must first find the secondary-market price of the bond p^{s_t}.

Lemma 1. *The secondary-market price of the bond is* $p^{s_t} = \eta\mathbb{E}_t[v^{s_{t+1}}]$.

The bond price splits the gains from trade between H_t and C_t in proportions η and $1-\eta$. Since H_t has value zero in this case (H_t dies at the end of the period and the bond is not demandable), the gains from trade are just the value $\mathbb{E}_t[v^{s_{t+1}}]$ of the bond to the new debtholder C_t.

By the preceding lemma, Eq. (6) gives the value in each state $s_t \in \{1, 0\}$:

$$v^1 = \frac{\rho R + \lambda(1-\rho)\Big(1-\theta(1-\sigma^1\eta)\Big)v^0}{\rho + (1-\rho)\Big(\lambda + (1-\lambda)\theta(1-\sigma^1\eta)\Big)} \tag{7}$$

and

$$v^0 = \frac{\rho R}{\rho + (1-\rho)\theta(1-\eta\sigma^0)}, \tag{8}$$

having used that $\mathbb{E}[v^{s_{t+1}} \mid s_t = 1] = \lambda v^0 + (1-\lambda)v^1$ and $\mathbb{E}[v^{s_{t+1}} \mid s_t = 0] = v^0$.

We now compare the bond's debt capacity (Eq. (7) with $R = y$ and $\sigma^1 = \sigma^0 = 1$)[12] to the puttable loan's (Eq. (4) with $R = y$ and $r = \ell$):

Proposition 2. (Benchmark: tradeability substitutes demandability.) *Suppose the bond circulates in equilibrium ($\sigma^0 = \sigma^1 = 1$).*[13]

If

$$\frac{\rho y + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta} < c \le \frac{\rho y}{\rho + (1-\rho)\theta(1-\eta)}, \tag{9}$$

then B can fund itself with a bond but not with a puttable loan (or a loan).

If the bond circulates, B can borrow against the full value y whenever H_t has all of the bargaining power. I.e. if $\sigma^1 = \sigma^0 = 1$, then there is no role for demandability whenever $\eta \to 1$. Hence, the analysis so far supports Jacklin's (1987) intuition that tradeability substitutes for demandability.[14] If C_0 is hit by a liquidity shock, he can trade B's debt in the market, rather than die with it. In other words, like the option to demand, the option to trade insures C_0 against bad outcomes, making him more willing to lend. Moreover, absent trading frictions ($\eta \to 1$), B can expand its debt capacity more by issuing tradeable debt (a bond) than by issuing demandable debt (a puttable loan). However, we will see next that with trading frictions ($\eta < 1$), there is a role for demandability, even if debt is never redeemed in equilibrium.

4. Banknote: demandability, fragility, suspension, and banking

In this section, we analyze the banknote and present our main results.

4.1. The two sides of demandability

Now we consider a banknote, i.e. tradeable, demandable debt. At Date t, the value v^{s_t} of the banknote can be written recursively:

$$v^{s_t} = \rho R + (1-\rho)\Big(\theta\big(\sigma^{s_t}p^{s_t} + (1-\sigma^{s_t})r\big) + (1-\theta)\mathbb{E}_t[v^{s_{t+1}}]\Big). \tag{10}$$

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The terms are determined as follows. With probability ρ, B's investment pays off and B repays R. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder H_t is hit by a liquidity shock. Since the banknote is both tradeable and demandable, H_t gets p^{s_t} if he finds a counterparty, which happens with probability σ^{s_t}, and otherwise redeems on demand and gets r. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains the banknote at Date $t+1$, which has value $\mathbb{E}_t[v^{s_{t+1}}]$ at Date t since there is no discounting.

To solve for the value v^{s_t}, we must first give the secondary-market price of the banknote p^{s_t}.

Lemma 2. *The secondary-market price of the banknote is* $p^{s_t} = \eta\mathbb{E}_t[v^{s_{t+1}}] + (1-\eta)r$.

The price of the banknote splits the gains between H_t and C_t in proportions η and $1-\eta$. Since H_t has value r (H_t redeems on demand and gets r if he does not trade with C_t), the gains from trade are $\mathbb{E}_t[v^{s_{t+1}}] - r$, the value to the new debtholder C_t minus the value to the current debtholder H_t. The price that splits these gains is $p^{s_t} = r + \eta(\mathbb{E}_t[v^{s_{t+1}}] - r) = \eta\mathbb{E}_t[v^{s_{t+1}}] + (1-\eta)r$.[15]

By the preceding lemma, Eq. (10) gives the value in each state $s_t \in \{1, 0\}$:

$$v^1 = \frac{\rho R + (1-\rho)\Big(\lambda\big(1-\theta(1-\eta\sigma^1)\big)v^0 + \theta(1-\eta\sigma^1)r\Big)}{\rho + (1-\rho)\Big(\lambda + (1-\lambda)\theta(1-\eta\sigma^1)\Big)} \tag{11}$$

and

$$v^0 = \frac{\rho R + (1-\rho)\theta\big(1-\eta\sigma^0\big)r}{\rho + (1-\rho)\theta\big(1-\eta\sigma^0\big)}, \tag{12}$$

having used that $\mathbb{E}[v^{s_{t+1}} \mid s_t = 1] = \lambda v^0 + (1-\lambda)v^1$ and $\mathbb{E}[v^{s_{t+1}} \mid s_t = 0] = v^0$. The expressions above point to the value of the banknote v^{s_t} being increasing in r (all else equal), as summarized in the next proposition:

Proposition 3. *(Bright side of demandability.) Conditional on circulation (i.e. on $\sigma^{s_t} = 1$ for all s_t), increasing the redemption value r increases the value of the banknote.*

The result captures the bright side of demandability. Even if the banknote circulates until maturity, the option to redeem on demand is valuable, especially for high r. The reason is that the option, even if never exercised, puts the debtholder H_t in a strong bargaining position with his counterparty, increasing the price he can sell the debt for. Thus, given secondary market trading frictions ($\eta < 1$), demandability *complements* tradeability:[16] your option to demand debt increases the price you trade at, something we refer to as "the price effect of demandability." The effect leads to a high debt capacity in the primary market: in anticipation of being able to sell at a high price in the secondary market, C_0 is willing to pay a high price in the primary market.[17] As a result, demandable debt can help B get its investment off the ground:

Corollary 1. *Suppose the banknote circulates ($\sigma_t = 1$ for all t).*

If

$$\max\left\{\frac{\rho y + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta}, \frac{\rho y}{\rho + (1-\rho)\theta(1-\eta)}\right\} < c \le \frac{\rho y + (1-\rho)\theta(1-\eta)\ell}{\rho + (1-\rho)\theta(1-\eta)}, \tag{13}$$

then B can fund itself only with the banknote.

The left-most term in Eq. (13) captures the debt capacity of the puttable loan—a banknote that cannot be traded—and a bond—a banknote that cannot be demanded. The right-most term is the debt capacity of the banknote itself—which is both demandable and tradeable. The result is illustrated in Fig. 3, which plots the regions in which financing via each instrument is feasible.

The redemption value can affect not only the value of the banknote conditional on circulation, but also whether it circulates in the first place. A banknote with terminal repayment R and redemption value r (we ultimately determine R and r in equilibrium in Proposition 5) circulates at Date t if C_t prefers to enter at cost k, paying p^{s_t} for debt worth $\mathbb{E}_t[v^{s_{t+1}}]$, than to stay out. I.e.

$$\mathbb{E}_t[v^{s_{t+1}}] - p^{s_t} \ge k \tag{14}$$

[15] This result depends on how outside options determine the division of surplus in bargaining. See Section 5 for a discussion.

[16] Other papers show that there may still be a role for demandability if tradeability is limited (Allen and Gale, 2004; Antinolfi and Prasad, 2008; Diamond,

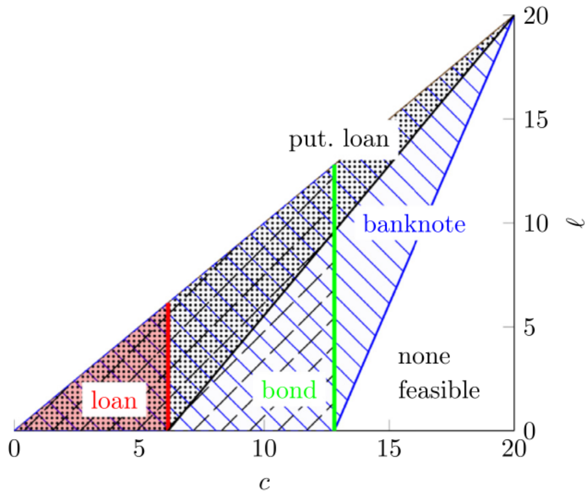

Fig. 3. Feasible Instruments. The banknote is feasible in the light blue shaded area; the bond in the green striped; the loan in the red hatched; the puttable loan in the gray dotted; and none in the white. (The northeast half is omitted as it violates the restriction that $\ell < c$.) The numbers used to make the plot are $\theta = 1/4$, $\rho = 1/10$, $y = 20$, and $\eta = 3/4$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Note: We think about these as annual numbers. $\theta = 1/4$, the number used in Ennis and Keister (2003), implies creditors suffer liquidity shocks on average once every four years. $\rho = 1/10$ implies the investment is long-term, taking ten years to complete on average. Given this maturity, $y = 20$ and $c = 10$ imply the investment has an annual return of 7.2%. $\eta = 3/4$ implies that debtholders get most of the surplus, but far from all of it; this is intended to capture some degree of competition among counterparties.

or, substituting for p^{s_t} from Lemma 2,

$$\mathbb{E}_t[v^{s_{t+1}}] - r \geq \frac{k}{1-\eta}. \tag{15}$$

This expression points to how increasing r can reduce the likelihood that the banknote circulates, as formalized in the next proposition:

Proposition 4. *(Dark side of demandability.) Increasing the redemption value r makes circulation "harder" in the following senses: there is a smaller set of entry costs k such that*

(i) *each counterparty C_t enters (given the strategy of other counterparties);*
(ii) *$\sigma^{s_t} = 1$ for all s_t is an equilibrium of the $t > 0$ subgame;*
(iii) *$\sigma^{s_t} = 0$ for all s_t is not an equilibrium of the $t > 0$ subgame.*

This result reveals that demandability can have a dark side. The same boost in price that benefits H_t (the seller) harms C_t (the buyer) thereby making him reluctant to enter, and if C_t does not enter, the note does not circulate.

Overall, demandability cuts both ways. It is both the thing that allows B to fund itself and the thing that could prevent its debt from circulating. It increases B's debt capacity, since it props up the price of B's debt (Proposition 3). But it also increases B's liquidation risk, making circulation harder (Proposition 4).

4.2. Money runs in equilibrium

We now turn to characterizing an equilibrium in which B borrows via a banknote that circulates initially, but fails to circulate in a confidence crisis:

$$\sigma^{s_t} = \begin{cases} \sigma^1 & \text{if } s_t = 1, \\ \sigma^0 & \text{if } s_t = 0. \end{cases} \tag{16}$$

I.e. C_t does not enter in a crisis, leading H_t to redeem on demand if (and only if) shocked, something we refer to as a "money run" as it is the result of the failure of B's debt to circulate "as money." The next proposition fully characterizes the equilibrium, which we refer to as a "sunspot-run equilibrium," and we focus on for much of the paper.

Proposition 5. *(Equilibrium with sunspot runs.) Suppose that the condition (13) is satisfied strictly. As long as λ is sufficiently small, there exists $\underline{k}$ and $\bar{k}$ (given explicitly in the Eqs. (B.52) and (B.53) in the proof) such that as long as $\underline{k} < k < \bar{k}$, B can fund its investment only with tradeable, demandable debt (a banknote), even though it admits a money run when $s_t = 0$. Specifically,* *the value of the banknote when* $s_t = 0$ *is*

$$\nu^0 = \frac{\rho R + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta}, \tag{18}$$

the repayment R is

$$R = c + \frac{(1-\rho)\theta\Big(\rho\big(\lambda + (1-\lambda)(1-\eta)\big) + (1-\rho)\big(\lambda + (1-\lambda)\theta(1-\eta)\big)\Big)}{\rho\Big(\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)\Big)}\Big(c - \ell\Big), \tag{19}$$

and the redemption value is $r = \ell$.

The result says that B chooses to set the maximum redemption value, $r = \ell$. The reason is that it allows it to borrow more cheaply (at lower R), due to the bright side of demandability (Proposition 3). By increasing C_0's resale value, it makes him willing to lend c at a lower price.[18]

But, as per the dark side of demandability (Proposition 4), the banknote can fail to circulate, leading to a money run. Unlike the bank-run literature, which has stressed bank failures resulting from shocks to fundamentals (e.g., Allen and Gale, 1998 and Gorton, 1988) or beliefs about primary market withdrawals (Diamond and Dybvig, 1983), we connect banks with the circulation of their liabilities. We are not the first to make this connection; e.g., Friedman and Schwartz (1963) suggest that the bank failures disrupt economic activity because they create liquidity necessary for economic activity. The twist here is that the chain of causation goes in the opposite direction: the bank fails only because the banknote fails to circulate (and is thus redeemed on demand). Thus, a run can occur even with a single creditor, who redeems his debt when he cannot trade it. It need not be the result of many creditors racing to be the first to redeem from a common pool of assets.

The result above also provides closed-form expressions that make it is easy to see how the price of debt depends on parameters:

Corollary 2. *(Comparative statics.) The (net) interest rate* $(R-c)/c$ *is*

(i) *decreasing in the liquidation value* ℓ*;*
(ii) *decreasing in debtholders' bargaining power* η*;*
(iii) *decreasing in creditors' liquidity horizon* $1/\theta$*;*
(iv) *increasing in the probability of a confidence crisis* λ*;*
(v) *increasing in the investment size* c*;*
(vi) *increasing in the investment horizon/expected maturity* $1/\rho$*. Moreover, the term structure is upward sloping, in the sense that the yield*[19] $\rho(R-c)/c$ *is also increasing in* $1/\rho$*.*

In our model, the interest rate is compensation for liquidity risk. The results (i)–(iv) capture that increasing ℓ and η decrease liquidity risk and increasing θ and λ increase it. (v) says that bigger investments are effectively riskier (all else equal). The reason is that, for fixed liquidation value ℓ, they are liquidated at a larger discount in a confidence crisis. (vi) says that longer maturity investments demand not only higher repayments, but also higher per-period interest rates, even though there is no discounting in preferences. The reason is that as maturity increases both the probability that C_0 has to trade at a discount before maturity *and* the size of the discount he trades at increase. So illiquidity in the secondary market generates the term structure.[20]

4.3. Suspension of convertibility

One policy regulators use to put a stop to runs is suspension of convertibility: B cannot redeem on demand, at least temporarily. We explore what such a policy would do here.

We model (partial) suspension contingent on the confidence crisis: if C_t demands redemption given $s_t = 0$, B's pays its debt with probability $1 - \Sigma$ and does not with probability Σ. Σ reflects the degree of suspension; $\Sigma = 0$ is the unregulated environment and $\Sigma = 1$ is full suspension. Such random suspension could be due to a government's ex ante precommitment to an intervention probability or uncertainty about whether it will choose to intervene at all ex post. It is only players' beliefs about future anticipation that matter for the equilibrium.

We find that an aggressive suspension policy can potentially not only prevent runs, but also restore circulation.

[18] Note that the payoff y appears only via condition (13) that ensures that B's debt capacity is greater than the investment cost c. R, r, and ν^{s_t} all depend on c but not on y. The reason is that when B makes the offer at Date 0, it takes only C_0's break-even condition into account, which depends on c but not on y. (In fact, if $\lambda = 0$, then $\nu^1 = c$; cf. Proposition 7.) If, on the other hand, C_0 made the offer, it would take only B's break-even condition into account,

Proposition 6. *(Suspension of convertibility.) Define*

$$\Sigma^{*} := \frac{(1-\eta)\Big(\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta(1-\eta)\big)\Big)(\ell - c) + \Big(\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)\Big)k}{(1-\eta)\big(\rho + (1-\rho)\lambda\big)\ell}. \tag{20}$$

An aggressive suspension policy $\Sigma > \Sigma^*$ both restores circulation and prevents runs in the sense that there is no equilibrium in which the banknote does not circulate in the confidence crisis.

If inequality (9) holds, the banknote is more valuable in the run-free equilibrium with suspension ($\Sigma > 0$) than in the sunspot-run equilibrium without it ($\Sigma = 0$).

Intuitively, if convertibility is suspended, the debtholder H_t can no longer redeem his note from the bank if trade fails. This decreases his outside option when bargaining with his counterparty C_t, putting C_t in a relatively strong bargaining position. If suspension is sufficiently aggressive, this makes C_t's bargaining position so strong that he is willing to enter even if he believes no one else will enter in the future. This restores circulation. If suspension is not sufficiently aggressive, however, C_t's bargaining position may not be strong enough to induce him to enter. This decreases the value of the banknote to H_t, making him less willing to buy the debt in anticipation of suspension and thereby decreasing what B can borrow. Overall, this suggests that a regulator aiming to prevent runs, restore liquidity, and preserve access to credit should treat suspension as a blunt instrument.

4.4. Banking

We now suppose that the horizon mismatch (Eq. (13)) is so severe that the borrower cannot raise c to fund its investment, not even via a banknote. In this case, direct finance is not possible. But perhaps a form of intermediated finance is?

To address this question, we now consider N parallel versions of our baseline model: N identical borrowers $B^1, \ldots, B^N$ can do parallel investments at Date 0, and N identical creditors $C_t^1, \ldots, C_t^N$ can enter parallel markets at each Date $t > 0$.

At Date 0, the borrowers can issue mutualized instruments, backed by the whole pool of their investments. If any debtholder redeems at Date t, he is paid out of the liquidation value. If the total liquidation value is not sufficient to pay redeeming debtholders, they are paid pro rata and others get nothing; otherwise, redeeming debtholders are paid in full and others' claims are reduced pro rata. The protocol by which the liquidation value is distributed matters only in so far as redeeming debtholders get paid ahead of others (this also holds, e.g., with sequential service à la Diamond and Dybvig, 1983). Therefore, if only one debtholder redeems it has a claim on the whole liquidation value $N\ell$—the constraint $r \le \ell$ in the baseline model need not be satisfied.

At each Date $t > 0$, each version of the model proceeds exactly as in the baseline, as described in Section 2. Note that we assume that the parallel versions of the model are identical in every state, i.e. investments/liquidity shocks are perfectly correlated across borrowers/creditors. Thus, there are no diversification benefits from pooling loans/deposits as in Diamond (1984)/Diamond and Dybvig (1983).

We find that even absent diversification, the borrowers can benefit from pooling their investments to raise funds as they can give each debtholder a claim on the whole liquidation value by setting $r = N\ell$. This is not valuable if all redeem, in which case each gets only ℓ, namely a pro rata share of the total liquidation value $N\ell$. But it is valuable if *none* redeems. Why?

The answer is that in an equilibrium in which banknotes circulate, no one redeems on the equilibrium path; thus, if one debtholder deviates, he is the only one redeeming, and can get paid up to $N\ell$.[21] As per the bright side of demandability, a high redemption value r is valuable, even if the redemption option is never exercised. Economies of scale emerge as a result. The value of the redemption option for N borrowers together exceeds N times the maximum that each could offer, as the option to redeem can be enjoyed by each creditor without making it unavailable to others. As such, increasing r can benefit all N debtholders simultaneously—it is "non-rivalrous."

As the number of borrowers N pooling assets increases, $N\ell$ becomes arbitrarily large. But, as per the dark side of demandability, it can be still limited by creditors' entry condition: if it is too large, they anticipate being in a weak bargaining position and do not enter—banknotes do not circulate. I.e. the entry condition (Eq. (15)) puts an upper bound $r^{\max}$ on r:

$$r \le r^{\max} := \mathbb{E}_t[v^{s_{t+1}}] - \frac{k}{1-\eta}. \tag{21}$$

(This constraint is slack by construction in our baseline equilibrium, as we focus on parameters for which the banknote can circulate even if $r = \ell$ (Proposition 5).)

Increasing r to $r^{\max}$ increases the borrowers' debt capacity (Corollary 1), which is N times the value of each banknote, computed by setting $r = r^{\max}$, $R = y$, and $\sigma^1 = \sigma^0 = 1$ in the value of the banknote in Eq. (11):

where the second term above equals the total expected entry costs of trading a banknote.[22]

Now, the condition that borrowers can get their investments off the ground—that the total debt capacity exceeds the total funding need, $N \max v \geq Nc$—can be re-written as

$$y - c - \text{total expected entry costs} \geq 0, \tag{23}$$

which says that, by forming a "bank," the borrowers can issue banknotes to fund all (and only) investments with positive total surplus, defined as the NPV net of the total expected entry costs:

Proposition 7. *(Banking.) Suppose*

$$N \geq \frac{1}{\ell}\left(y - \frac{\rho + (1-\rho)(1-\eta)}{\rho(1-\eta)}k\right). \tag{24}$$

There is an equilibrium in which borrowers raise Nc by issuing a banknote to each of the Date-0 creditors if and only if the investments have positive total surplus, i.e. if their NPV is higher than the total expected entry costs, or

$$y - c \geq \frac{(1-\rho)\theta}{\rho}k. \tag{25}$$

In equilibrium, the value of the banknote is

$$v = \frac{\rho R + (1-\rho)\theta r}{\rho + (1-\rho)\theta} = c, \tag{26}$$

the repayment R is

$$R = c + \frac{(1-\rho)\theta(1-\eta)}{\rho}(c - r), \tag{27}$$

and the redemption value is

$$r = r^{\max} := R - \frac{\rho + (1-\rho)\theta(1-\eta)}{\rho(1-\eta)}k = c - \frac{k}{1-\eta}. \tag{28}$$

By setting $r = r^{\max}$, the borrowers can decrease their borrowing costs and fund all positive-surplus investments. But setting r so high makes counterparties indifferent between entering and staying out. This could make borrowers especially susceptible to runs, in the sense that an arbitrarily small change in a counterparty's entry cost or even belief about others' strategies makes him stay out, leading to a money run. (Such a change could be a sunspot event as in Proposition 5, or even unanticipated.)

And now a money run has even more severe consequences. As in a real-world bank run, there is mass liquidation: with $r > \ell$, multiple investments need to be liquidated to redeem each banknote. In addition to this fragility, the coalition of borrowers has other defining features of a real-world bank.

1. **Liquidity transformation.** The bank funds illiquid assets (non-tradeable investments that are costly to liquidate early) with liquid liabilities (circulating demandable debt).
   - Issuing liquid (tradeable) liabilities gives creditors insurance against liquidity shocks.
2. **Maturity transformation.** The bank funds long-term investments with short-term (demandable) liabilities.
   - Issuing demandable liabilities allows creditors to trade at a high price given liquidity shocks.
3. **Asset pooling.** The bank pools borrowers' investments, reusing their liquidation value to back demandable debt.
   - Issuing debt backed by a pool of assets gives creditors a high redemption value.
4. **Dispersed depositors (creditors).** The bank borrows from a large number of dispersed creditors.
   - Issuing debt to many creditors gives them the option to redeem against the same assets (hence dispersed creditors are necessary for asset pooling to help).
5. **Fragility.** The bank borrows via debt that is susceptible to runs, and runs force early liquidation of multiple investments.
   - Issuing run-prone debt, i.e. demandable debt with high redemption value, is necessary to make the secondary market price high enough that the bank can fund efficient investments.

The banking equilibrium also makes it easier to apply our model to contemporary deposit markets, in which entry costs often seem to be small and deposits are redeemable at par:

Corollary 3. *Consider the banking equilibrium in Proposition 7 in which $r = r^{\max}$. As entry costs become small, i.e. $k \to 0$, the*

Here, unlike in the baseline model (Proposition 5), a run could occur no matter how small the entry cost k is. If k is small, the borrowers make r high, so counterparties are still indifferent between entering and staying out (cf. Eq. (21)):[23]

Corollary 4. *(Small- k runs.) If C_t's beliefs change from $\sigma_{t'} = 1$ to $\sigma_{t'} = 0$ for $t' > t$, C_t does not enter (no matter the entry cost k).*

The flip side of the last result is that for $k > 0$, we should expect to see a penalty for early withdrawal (i.e. $r < R$), in line with those we see for less liquid forms of bank debt, such as savings accounts and certificates of deposit.

5. Discussion of assumptions

Entry costs. As we stress in Section 4.4, money runs can arise no matter how small k is. Whether k reflects the literal cost of entering a trade or the opportunity cost of the time taken in trading, such a small k could be realistic for some contemporary markets, like retail markets in which consumers trade deposits for goods via debit cards. But a larger entry cost also has natural interpretations. Historically, it could represent the physical/temporal costs of coming to market or, alternatively, of acquiring the expertise/technology to check for counterfeit instruments. More generally, it could represent any relative cost of searching for a counterparty as in the search money literature, of trading/transacting as in the finance literature, or of posting a vacancy as in the labor literature. Any cost sunk before counterparties meet suffices for our results.

Rollover. To focus on trade in the secondary market, we want to abstract from rollover in the primary market (its practical importance notwithstanding). Indeed, the entry-bargaining-settlement setup in Section 2.3 deliberately precludes strategies in which B borrows via one-period contracts, and issues new debt to C_t to settle its existing debt with H_t at each date: since B would have to settle first with C_t and then with H_t, this would require an additional settlement stage. Moreover, such a one-period rollover strategy would typically be less desirable than demandable debt in our baseline environment anyway: someone would have to pay the cost k to enter and buy the new issue in every period, rather than to enter and trade existing debt only in periods in which H_t is hit by a liquidity shock. More practically, secondary market trade allows the borrower to avoid floatation costs, which could be prohibitive if borne in every period in the rollover strategy.

That said, below we include rollover in our environment (under some additional assumptions), and show that money runs can still occur (Section 6.3).

Bargaining protocol. In our model, demandability matters because the redemption value r serves as the outside option in bargaining. Thus, security design can substitute for market design: the borrower can adjust the terms of trade in the secondary market, choosing r to calibrate the division of surplus between counterparties, even though the bargaining power η is immutable.

Our results hold for bargaining protocols, like Nash bargaining, in which the outside option determines the division of the surplus. Not every non-cooperative bargaining game has this feature in equilibrium (Sutton, 1986). But many do. Indeed, the Nash outcome coincides with the equilibrium of a game in which bargainers either (i) risk having the bargaining process suddenly break down or (ii) have the ability to make take-it-or-leave-it offers (see, e.g., Binmore et al., 1986). Within our model, the risk of a breakdown could reflect the probability that a counterparty abandons the negotiation because he is hit by a liquidity shock himself or because he finds another, more profitable trade to execute. And the ability to make take-it-or-leave-it offers could reflect the situation in modern "'hi-tech' markets in which binding deals are made quickly over the telephone [or Bloomberg chat]" (Binmore et al., 1992, p. 190; see Shaked, 1994).

Infinite horizon. Money runs arise due to dynamic coordination—a counterparty enters if he believes his future counterparty will, who enters if he believes his future counterparty will.... Thus, if everyone knows that any counterparty is the last one, he will never enter, and the "good" equilibrium will unravel by backward induction. We avoid this by assuming that the horizon is infinite, so every counterparty has a future counterparty. Indeed, there is no date at which the banknote expires for sure; as such, tradeable, demandable debt may have more in common with perpetual debt than with near-dated fixed-term debt.

The infinite horizon is one way to capture the idea that each counterparty believes that an instrument could continue for one more period with positive probability. It is the way used in the new monetarist literature, following Kiyotaki and Wright (1989,1993), but it is not the only way; for example, counterparties could be uncertain about their position in a finite trading sequence (see, e.g., Moinas and Pouget, 2013).

6. Extensions

In this section, we analyze extensions.

Throughout, we focus on stationary equilibria, so we can drop the state-dependence in the superscripts: $\nu^{s_t} = \nu$, $\sigma^{s_t} = \sigma$,

6.1. Random entry costs

We now extend the model so C_t's entry cost is a random variable $\tilde{k}_t$, so that C_t enters only if his entry cost is below a threshold k^*.

We assume that the distribution of entry costs is Pareto, with support $[k_0, \infty)$ and exponent one, $\tilde{k}_t \sim \text{Pareto}(k_0, 1)$. This allows us to solve for cut-off equilibria in closed-form. To do so, we suppose that C_t believes that future creditors $C_{t'}$ enter whenever $k_{t'} \leq k^*$, so his entry condition is

$$v - p = \frac{\rho\left(R - r\right)}{\rho + (1-\rho)\theta\left(1 - \eta\,\mathbb{P}\left[\,\tilde{k}_{t'} \leq k^*\,\right]\right)} \geq \frac{k_t}{1-\eta}. \tag{29}$$

This is the entry condition (B.42) with future creditors' entry probability given by the probability their entry cost exceeds the cut-off k^*, rather than by their mixing probability σ. Using the Pareto distribution to substitute in for this probability, $\mathbb{P}\left[\,\tilde{k}_{t'} \leq k^*\,\right] = 1 - k_0/k^*$, gives the next proposition:

Proposition 8. *(Cut-off equilibrium with random entry costs.) Define*

$$k^* = \frac{\rho(1-\eta)(R-r) - (1-\rho)\theta\eta k_0}{\rho + (1-\rho)\theta(1-\eta)}. \tag{30}$$

If entry costs follow the Pareto distribution described above and $k^* > k_0$, then there is a unique stationary cut-off equilibrium of the $t > 0$ subgame in which C_t enters if and only if $k_t \leq k^*$.

Observe that, in this equilibrium, H_t withdraws with positive probability on the equilibrium path, namely whenever he suffers a liquidity shock and his counterparty C_t does not enter. C_t enters only if $k_t \leq k^*$, or

$$\mathbb{P}\left[\tilde{k}_t \leq k^*\right] = 1 - \frac{k_0}{k^*} = 1 - \frac{\left(\rho + (1-\rho)\theta(1-\eta)\right)k_0}{\rho(1-\eta)(R-r) - (1-\rho)\theta\eta k_0}. \tag{31}$$

6.2. Asset choice

What if a single borrower B chooses the type of its investment before borrowing from C_0? Do frictions in the secondary market distort its choice? Yes, toward high-liquidation-value investments:

Proposition 9. *(Excessive liquidity.) Suppose that B can choose between an investment with payoff y and liquidation value ℓ and another investment that is otherwise identical but has lower payoff $y' < y$ and higher liquidation value ℓ', where*

$$\ell' > \ell + \frac{\rho}{(1-\rho)\theta(1-\eta)}\left(y - y'\right). \tag{32}$$

There exists an investment cost c such that in any equilibrium in which investment occurs, B chooses the low-NPV, high-liquidation-value investment (y', ℓ').

Intuitively, with a high-liquidation-value investment, B can issue a high-redemption-value banknote and borrow more. Thus, to make its debt money-like, B chooses to increase its liquidation value even at the expense of NPV.

6.3. Partial rollover

We now turn to a version of the model in which counterparties are matched with debtholders in a single market via a homogenous matching technology. (This differs from Section 4.4, in which they trade in parallel markets.) This set-up allows us to show that money runs can occur even if (i) there are no aggregate shocks to liquidity and (ii) B can raise money from new creditors at the beginning of each date, thereby rolling over its debt to meet redemptions.[24] Moreover, unlike in the baseline model, not every withdrawal is a run. Rather, some debtholders redeem at each date.

Here we do not model funding/investment, but focus on the secondary market, assuming that banknotes are held by a unit continuum of debtholders, a fraction θ of which needs liquidity at each date. Counterparties can enter at cost k, in which case they are matched with debtholders via a homogenous matching function. Thus, the probability σ with which a debtholder meets a counterparty depends on the number of counterparties who enter. The fraction $\theta\sigma$ of debtholders who meet counterparties trade in the secondary market. The remaining $\theta(1-\sigma)$ redeem for r. We assume that B issues new identical banknotes to raise exactly enough to meet these redemptions at the beginning of each date.[25]

The next result says that this set-up has multiple steady state equilibria. Indeed, there is a "good equilibrium," in which many counterparties enter and few debtholders are left unmatched. In this equilibrium, there are relatively few withdrawals at each date, so B chooses its rollover strategy to raise a relatively small amount of liquidity. But there is also a "bad equilibrium," in which few counterparties enter and many debtholders are left unmatched. In this equilibrium, there are more withdrawals at each date, so B has to choose a rollover strategy to raise more liquidity. Thus, a change in beliefs can lead to a money run analogous to that in Proposition 5: if counterparties today believe that few of their future counterparties will enter, then few of them enter today; this leads to an unexpectedly high number of withdrawals—a money run.

Proposition 10. *(Money runs with partial rollover.) Let the matching technology be given by $\sigma = m\sqrt{q}$, where q is the number of counterparties that enter and $m > 0$ is a parameter. Suppose that B borrows via banknotes from a continuum of creditors. The $t > 0$ subgame has two stationary equilibria, one in which many counterparties enter,*

$$\sigma = \frac{k\big(\rho + (1-\rho)\theta\big) + \sqrt{k^2\big(\rho + (1-\rho)\theta\big)^2 - 4m^2 k\rho(1-\rho)(R-r)\theta\eta(1-\eta)}}{2k(1-\rho)\theta\eta} =: \sigma_+ \tag{33}$$

—banknotes are liquid—and another in which few counterparties enter,

$$\sigma = \frac{k\big(\rho + (1-\rho)\theta\big) - \sqrt{k^2\big(\rho + (1-\rho)\theta\big)^2 - 4m^2 k\rho(1-\rho)(R-r)\theta\eta(1-\eta)}}{2k(1-\rho)\theta\eta} =: \sigma_- \tag{34}$$

—banknotes are illiquid—as long as σ_+ and σ_- above are well-defined probabilities.

This result implies that money runs can occur even with no aggregate risk, no rollover risk, and no sequential-service constraint. This affirms that money runs result only from intertemporal coordination in the secondary market and helps distinguish our model of bank fragility from models of rollover risk (e.g., Acharya et al., 2011 and He and Xiong, 2012).

7. Related literature

We make three main contributions to the literature. First, we offer a new rationale for demandable debt (Proposition 3), complementing the literature that shows how demandability can help to mitigate moral hazard problems (Calomiris and Kahn, 1991 and Diamond and Rajan, 2001a, 2001b).[26] Our analysis also extends results in the literature on corporate bonds that suggest short-maturity bonds can have the benefit of high resale prices in the secondary market, but the cost of frequent debt issuances (Bruche and Segura, 2016 and He and Milbradt, 2014). These papers restrict attention to debt contracts as in Leland and Toft (1996). We point out that with more general contracts the benefit can come without the cost: demandable debt props up the price even though the option to redeem need never be exercised.[27] Second, we show how a bank run can result not from coordination among current depositors in the primary market, but between current and future depositors in the secondary market, thereby connecting bank fragility à la Diamond and Dybvig (1983) with liquid dry-ups à la Kiyotaki and Wright (1989,1993).[28] Third, we show that the need to create circulating demandable debt gives rise to numerous other banking activities. This adds to the literature on the foundations of banking, connecting pooling assets and dispersed liabilities (e.g., Boyd and Prescott, 1986; Diamond, 1984; Diamond and Dybvig, 1983; Ramakrishnan and Thakor, 1984) with liquidity creation (e.g., Andolfatto et al., 2019; Donaldson et al., 2018a; Gu et al., 2013).

8. Conclusion

What is a bank? A bank is something that creates liquidity. By thinking about a bank this way, we found a new rationale for demandable debt, a new type of bank run—a "money run"—and a new explanation for the other quintessential things banks do, such as pooling assets and maturity/liquidity transformation. The perspective matters for policy. Among other things, it suggests a benefit of suspension of convertibility which is new to the literature.

[26] In their conclusion, Diamond and Rajan (2001a) make the link between demandability and circulating banknotes informally, saying that

> deposits are readily transferable, and liquid, because buyers of deposits have no less ability to extract payment than do sellers of deposits. Thus, the deposits can serve as bank notes or checks that circulate between depositors. This could explain the special role of banks in creating inside money (p. 425).

We make this link formally in this paper.

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J.R. Donaldson and G. Piacentino Journal of Monetary Economics xxx (xxxx) xxx

Appendix A. Further Discussion of Policy and Applications

A1. Policy

Our model gives new perspectives on a number of other policies.

1. **Narrow banking.** In our model, a bank can fund all worthwhile investments if it can pool them and issue demandable debt backed by the whole pool (Proposition 7).
   This suggests a downside to the idea of narrow banking, which suggests that real investments should be separate from deposit-taking (its financial stability benefits notwithstanding).
2. **Asset purchase guarantees.** In 2008 the US Treasury opened its Temporary Guarantee Program, in which it promised to buy the shares of money market mutual funds at a guaranteed price. This off-equilibrium promise to buy money-like securities could eliminate the "bad" equilibrium in our model, in which counterparties do not enter the secondary market, fearing it will dry up in the future.
3. **Capital requirements.** In the context of our model, capital requirements could be a double-edged sword. They can help, by curbing banks' incentive to use too much demandable debt (Proposition 4). But they can also hurt, by inefficiently constraining investment (Proposition 3).

A2. Applications

One way banks create liquidity is by providing a payment instrument to help solve the double-coincidence problem—to help you buy things that are hard to get by barter (see, e.g., Kiyotaki and Wright, 1989; Kiyotaki and Wright, 1993; Wicksell, 1907). I.e. banks create money.[29] Thus our model could speak to monetary fragility generally. However, we have abstracted from some things stressed in contemporary discussions of money in connection with banking and financial stability. Notably, (i) we assume that all money is created by banks, abstracting from outside money and assuming that debt is redeemable for real goods. (ii) We assume that there is a single bank, abstracting from interbank competition, trade, and clearing. We also make stark assumptions about the secondary market, modeling decentralized trade in the simplest way we can, with (iii) costly entry/trade and (iv) prices determined by bilateral bargaining.

In the Free Banking Era, our model assumptions seem to be mostly satisfied. Before Greenbacks were introduced in 1861, (i) all paper money was created by banks, and it was redeemable for gold and silver. Sometimes, (ii) there was only one bank in a geographical region (see, e.g., Helderman, 1931). As a result, counterparties would trade largely that bank's notes. (iii) Bilateral trade was costly for a number of reasons, including spatial separation, and the fact that "[e]ach time a transaction took place the seller [of goods] had to make some judgment about the quality of the particular set of bank notes being offered... [and the] process of making this judgment used real resources" (Rockoff, 1974, p. 144). Moreover, banknotes traded at different discounts, which varied depending on who traded and where, reflecting that (iv) prices were determined bilaterally. Indeed, notes tended to trade at higher prices when redemption values were higher and when physical redemption was cheaper (i.e. the issuer was physically closer), consistent with the effect of the redemption value r on the secondary market price through bargaining (see, e.g., Gorton, 1996 and Weber, 2005). However, the mapping from r to the price was not one-to-one, consistent with the effect of bargaining on the division of surplus (for $\eta \in (0, 1)$). And it was not homogenous across notes and time, consistent with the effect of a self-fulfilling aspect to prices, as in our model.

More often, however, our model assumptions are not satisfied so literally. But we think our model still speaks to these circumstances, albeit with a broader interpretation. (i) Central banks create paper and electronic money, which is what bank debt can be redeemed for. In this case, the good in our model should be interpreted as outside money, which can be used freely for consumption/investment. Typically, (ii) there are multiple banks in a geographical region. In this case, the bank in our model should be interpreted as the banking system, and rejecting a bank's notes as not accepting transfers from it. For example, in the recent crises in Argentina and Greece, merchants sometimes refused transfers from the domestic banking system, but accepted foreign ones. These crises also highlight that although (iii) trading costs and (iv) bilateral negotiations are less salient in developed economies in normal times, they can quickly reemerge in crisis. This is consistent with our model, which suggests that notes trade at par for small k (Corollary 3).

Indeed, many historical panics have features of money runs. For example, when merchants refused bank-drawn bills of exchange in 18th-century London, it led to the crisis of 1772[30]; when the Second Bank refused state bank notes in the early 19th-century US, it led to the crisis of 1819[31]; when New York clearing houses refused bank trusts' checks in the early

[29] Our model abstracts from the double coincidence problem (as we assume utility is transferable), but mainly for simplicity; including it as in, e.g, Trejos and Wright (1995) would not change the main results (as the effect of bargaining power on entry would not be affected).

[30] See, e.g., Kosmetatos (2014) on how "[t]hrough drawers, acceptors, or endorsers of bills stopping [accepting them], [issuers]... quickly failed" in the

20th-century US, it led to the Panic of 1907[32]; when retailers refused checks in late 20th- and early 21st-century Argentina, it exacerbated both the banking panic of 1995 and the economic crisis of 1998–2002[33]; when retailers refused debit cards and wholesalers refused bank transfers in contemporary Greece, it exacerbated the Greek debt crisis.[34]

Appendix B. Proofs

B1. Proof of Proposition 1

For an instrument i, let $\max v_i$ be an instrument's debt capacity, i.e. its maximum value over any R, r, and σ^{s_t}:

$$\max v_i := \sup\left\{ v_i \,\middle|\, r \le \ell\,,\ R \le y\,, \sigma^{s_t} \in [0,1]\right\}. \tag{B.35}$$

So C_0 lends against instrument i only if $\max v_i \ge c$. Hence, B can fund itself with the puttable loan but not with the loan if and only if

$$\max v_{\text{loan}} < c \le \max v_{\text{putt. loan}}. \tag{B.36}$$

Substituting $r = \ell$ and $R = y$ into the expressions for their values in Eqs. (2) and (4) gives the condition in the proposition. □

B2. Proof of Lemma 1

When C_t and H_t are matched, H_t has been hit by a liquidity shock. Thus, C_t's value of the bond is $\mathbb{E}[v^{s_{t+1}}]$ and H_t's value of the bond is zero (since H_t consumes only at Date t and the bond is not demandable). The total surplus is thus $\mathbb{E}[v^{s_{t+1}}]$, which C_t and H_t split in proportions $1-\eta$ and η, in accordance with the Nash bargaining solution. Thus the price is $p^{s_t} = \eta\mathbb{E}[v^{s_{t+1}}]$. □

B3. Proof of Proposition 2

The proof is analogous to that of Proposition 1. B can borrow via a bond but not with a puttable loan if and only if

$$\max v_{\text{putt. loan}} < c \le \max v_{\text{bond}}, \tag{B.37}$$

where $\max v_i$ is as defined in Eq. (B.35). Substituting $r = \ell$, $R = y$, and $\sigma^1 = \sigma^0 = 1$ into the expressions for their values in Eqs. (4) and (7) gives the condition in the proposition. □

B4. Proof of Lemma 2

When C_t and H_t are matched, H_t has been hit by a liquidity shock. Thus, C_t's value of the banknote is $\mathbb{E}_t[v^{s_{t+1}}]$ and H_t's value of the banknote is r (since H_t consumes only at Date t, it redeems on demand if it does not trade). The gains from trade are thus $\mathbb{E}_t[v^{s_{t+1}}] - r$, which C_t and H_t split in proportions $1-\eta$ and η, in accordance with the Nash bargaining solution, i.e. p^{s_t} is such that

$$\text{H}_t \text{ gets} \quad \eta\left(\mathbb{E}_t[v^{s_{t+1}}] - r\right) + r = p^{s_t}, \tag{B.38}$$

$$\text{C}_t \text{ gets} \quad (1-\eta)\left(\mathbb{E}_t[v^{s_{t+1}}] - r\right) = \mathbb{E}_t[v^{s_{t+1}}] - p^{s_t}, \tag{B.39}$$

or $p^{s_t} = \eta\mathbb{E}_t[v^{s_{t+1}}] + (1-\eta)r$. □

B5. Proof of Proposition 3

The argument is in the text. □

Cobb describes it, "The merchant, the mechanic, the grocer, and the butcher began business in the morning... and their customers found that the bank note that passed freely yesterday was rejected this morning" (p. 36).

[32] See, e.g., Tallman and Moen (1990) on how "stop[ping] clearing checks for the Knickerbocker Trust Company" incited the 1907 panic; see also Frydman et al. (2015).

[33] See, e.g., the New York *Times* on how "[s]ome big businesses [were] demanding cash on delivery and refusing to accept checks" in the 1995 banking

B6. Proof of Corollary 1

The proof is analogous to those of Proposition 1 and Proposition 2. B can borrow via a banknote but not with a puttable loan or a bond if and only if

$$\max\left\{\max v_{\text{putt. loan}}, \max v_{\text{bond}}\right\} < c \le \max v_{\text{b.note}}, \tag{B.40}$$

where $\max v_i$ is as defined in Eq. (B.35). Substituting $r = \ell$, $R = y$, and $\sigma^1 = \sigma^0 = 1$ into the expressions for their values in Eqs. (4), (7), and (11) gives the condition in the proposition. □

B7. Proof of Proposition 4

We prove points (i)–(iii) in turn.

(i) Here we consider C_t's best response taking other counterparties' strategies as given, for $s_t = 1$ and $s_t = 0$ in turn.

- $s_t = 1$: By Eq. (15), C_t enters if and only if

$$\lambda v^0 + (1-\lambda)v^1 - r \ge \frac{k}{1-\eta}, \tag{B.41}$$

given $\mathbb{E}[v^{s_{t+1}} | s_t = 1] = \lambda v^0 + (1-\lambda)v^1$. Plugging in for v^1 from Eq. (11), the inequality above becomes

$$\frac{\lambda(v_0 - r) + (1-\lambda)\rho(R-r)}{\rho + (1-\rho)\Big(\lambda + (1-\lambda)\theta(1-\eta\sigma^1)\Big)} \ge \frac{k}{1-\eta}. \tag{B.42}$$

The LHS is decreasing in r (for fixed σ^1), given $v^0 - r$ is decreasing in r as we show next.

- $s_t = 0$: By Eq. (15), C_t enters if and only if

$$v^0 - r \ge \frac{k}{1-\eta}, \tag{B.43}$$

given $\mathbb{E}[v^{s_{t+1}} | s_t = 0] = v^0$. Plugging in for v^0 from Eq. (12), the inequality above becomes

$$\frac{\rho(R-r)}{\rho + (1-\rho)\theta(1-\eta\sigma^0)} \ge \frac{k}{1-\eta}. \tag{B.44}$$

The LHS is decreasing in r (for fixed σ^0).

(ii) There is an equilibrium in which $\sigma^0 = \sigma^1 = 1$ if and only if C_t enters given he believes all other counterparties do:

$$\mathbb{E}_t[v^{s_{t+1}}] - r\Big|_{\sigma^0=\sigma^1=1} \ge \frac{k}{1-\eta}, \tag{B.45}$$

or

$$\frac{\rho(R-r)}{\rho + (1-\rho)\theta(1-\eta)} \ge \frac{k}{1-\eta}. \tag{B.46}$$

The LHS is decreasing in r.

(iii) There is *no* equilibrium in which $\sigma^0 = \sigma^1 = 1$ if and only if C_t enters given he believes all other counterparties do *not*:

$$\mathbb{E}_t[v^{s_{t+1}}] - r\Big|_{\sigma^0=\sigma^1=0} \ge \frac{k}{1-\eta}, \tag{B.47}$$

or

$$\frac{\rho(R-r)}{\rho + (1-\rho)\theta} \ge \frac{k}{1-\eta}. \tag{B.48}$$

The LHS is decreasing in r. □

B8. Proof of Proposition 5

We first solve for the values v^0 and v^1 in terms of r and R given the strategies $\sigma^0 = 0$ and $\sigma^1 = 1$. We show that

Values. From Eq. (12) with $\sigma^0 = 0$, we have immediately that

$$v^0 = \frac{\rho R + (1-\rho)\theta r}{\rho + (1-\rho)\theta}. \tag{B.49}$$

From Eq. (11) with $\sigma^1 = 1$ and v^0 as above, we have immediately that

$$v^1 = \frac{\rho R + (1-\rho)\Big(\lambda\big(1-\theta(1-\eta)\big)v_0 + \theta(1-\eta)r\Big)}{\rho + (1-\rho)\Big(\lambda + (1-\lambda)\theta(1-\eta)\Big)}. \tag{B.50}$$

Best responses. By Eq. (15), $\sigma^1 = 1$ and $\sigma^0 = 0$ are best responses if

$$v^0 - r \;\le\; \frac{k}{1-\eta} \;\le\; \lambda v^0 + (1-\lambda)v^1 - r \tag{B.51}$$

This is satisfied for as long as $\underline{k} < k < \bar{k}$, where

$$\underline{k} := (1-\eta)(v^0 - r) \tag{B.52}$$

and

$$\bar{k} := (1-\eta)\Big(\lambda v^0 + (1-\lambda)v^1 - r\Big), \tag{B.53}$$

where v^0, v^1, and r are as given in the statement of the proposition.

B's utility. Below, we will use B's utility in state s_t, which we denote by u^{s_t} and can write recursively as

$$u^{s_t} = \rho(y-R) + (1-\rho)\Big(\theta\Big(\sigma^{s_t}\mathbb{E}_t[u^{s_{t+1}}] + (1-\sigma^{s_t})(\ell - r)\Big) + (1-\theta)\mathbb{E}_t[u^{s_{t+1}}]\Big). \tag{B.54}$$

The terms are determined as follows. With probability ρ, B's investment pays off and B repays R, keeping $y-R$. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder H_t is hit by a liquidity shock. With conditional probability σ^{s_t}, H_t finds a counterparty and B continues its investment, getting $\mathbb{E}_t[u^{s_{t+1}}]$, since there is no discounting. Otherwise, with conditional probability $1-\sigma^{s_t}$, H_t does not find a counterparty and redeems on demand. B must liquidate its investment and repay r, so it gets $\ell - r$. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. Again, B continues and gets $\mathbb{E}_t[u^{s_{t+1}}]$.

So, as $\sigma^1 = 1$ and $\sigma^0 = 0$,

$$u^1 = \frac{\rho(y-R) + (1-\rho)\lambda u^0}{\rho + (1-\rho)\lambda}, \tag{B.55}$$

having used that $\mathbb{E}[u^{s_{t+1}} \mid s_t = 1] = \lambda u^0 + (1-\lambda)u^1$ and $\mathbb{E}[u^{s_{t+1}} \mid s_t = 0] = u^0$.

Repayments. B maximizes $\mathbb{E}_0[u^{s_1}]$ subject to the constraint that C_0 funds c. As u^1 and u^0 are decreasing in the repayment R, it is determined by solving C_0's break-even condition:

$$c = \lambda v^0 + (1-\lambda)v^1. \tag{B.56}$$

Substituting in for v^0 and v^1 from Eqs. (B.49) and (B.50) and solving for R, we find

$$R = c + \frac{(1-\rho)\theta\Big(\rho\big(\lambda + (1-\lambda)(1-\eta)\big) + (1-\rho)\big(\lambda + (1-\lambda)\theta(1-\eta)\big)\Big)}{\rho\Big(\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)\Big)}\Big(c - r\Big), \tag{B.57}$$

which is the expression in the proposition if $r = \ell$.

To see that $r = \ell$, we can differentiate Eq. (B.55) above at $\lambda = 0$ to find:

$$\left.\frac{du^1}{dr}\right|_{\lambda=0} = \frac{1}{\rho + (1-\rho)\lambda}\left.\left(-\rho\frac{dR}{dr} + (1-\rho)\lambda\frac{du^0}{dr}\right)\right|_{\lambda=0} = \frac{(1-\rho)\theta(1-\eta)}{\rho} > 0, \tag{B.58}$$

given the expression for R in Eq. (B.57). I.e. u^1 is increasing in r for $\lambda = 0$. Given u^1 is uniformly continuous in λ, it is for small $\lambda > 0$ too. Thus B sets the maximum $r = \ell$ in this case.[35,36]

We can then use the expressions for R and v^0 above and substitute them into v_1 to find

$$v^1 = \frac{\Big(\rho + (1-\rho)\big(\lambda(1+\theta\eta) + (1-\lambda)\theta\big)\Big)c - (1-\rho)\lambda\theta\eta\ell}{\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)}, \tag{B.59}$$

as expressed in the proposition. □

B9. Proof of Corollary 2

The results follow directly from differentiation given the expression for R in Eq. (19). □

B10. Proof of Proposition 6

We first solve for the values v^0 and v^1 in terms of the redemption values, which are ℓ in normal times and $(1-\Sigma)\ell$ in confidence crises, given the strategies $\sigma^0 = 0$ and $\sigma^1 = 1$. We show that in this sunspot-run equilibrium, increasing Σ increases the repayment R required to fund c, and hence decreases debt capacity. Then we show that it might no longer be an equilibrium if Σ is above the threshold Σ^*. We describe when such an aggressive suspension policy increases debt capacity.

Values. Since suspension occurs only if $s_1 = 0$, v^1 is as above (Eq. (B.50) with $r = \ell$) and v^0 is as above (Eq. (B.49)) except with $r = (1-\Sigma)\ell$:

$$v^0 = \frac{\rho R + (1-\rho)\theta(1-\Sigma)\ell}{\rho + (1-\rho)\theta}. \tag{B.60}$$

Repayments. Now, the repayment R is determined by solving

$$c = \lambda v^0 + (1-\lambda)v^1. \tag{B.61}$$

Substituting in for v^0 and v^1 from Eqs. (B.60) and (B.50) and solving for R, we find

$$R = c + \frac{\theta(1-\rho)}{\rho}\,\frac{\lambda(c-(1-\Sigma)\ell) + \Big(\rho + (1-\rho)\theta\Big)(1-\eta)(1-\lambda)(c-\ell)}{\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)}. \tag{B.62}$$

This is increasing in Σ, implying that suspending convertibility can decrease debt capacity.

Is there a run in the crisis state? $\sigma^0 = 0$ is a best response if

$$v^0 - p^0 \;\le\; k \tag{B.63}$$

or, substituting,

$$\frac{\rho\Big(R - (1-\Sigma)\ell\Big)}{\rho + (1-\rho)\theta} \;\le\; \frac{k}{1-\eta}. \tag{B.64}$$

The inequality above binds if $\Sigma = \Sigma^*$ in Eq. (20). If $\Sigma > \Sigma^*$ it is violated. I.e. there is no run in equilibrium.

We now show that B can borrow more in the "new" equilibrium than in the "old" one as long as the inequality (9) holds. As the suspension policy takes effect only in the confidence crisis ($s_t = 0$), it suffices to compare the value of the banknote in the crisis under each policy.

- "Old equilibrium": in state 0 there is no suspension, but there is no circulation either, so the value of the banknote is the value of the puttable loan with effective redemption value $r = \ell$, which we know is lower than the value of a bond if inequality (9) holds.
- "New equilibrium": in state 0 there is suspension, but there is circulation, so the value of the banknote (which has effective redemption value $r = (1-\Sigma)\ell$) is bounded below by the value of the bond.

Overall, the value of the banknote with the suspension policy (in which case it circulates) exceeds its value without it (in which case there is a run).

B11. Proof of Proposition 7

Most of the argument is in the text preceding the proposition.

B12. Proof of Corollary 3

The result follows immediately from the expressions for $r^{\max}$ in Proposition 7 and p in Lemma 2 (given the expression for v in Proposition 7). □

B13. Proof of Corollary 4

In equilibrium, $r = r^{\max}$, implying C_t is indifferent between entering and staying out. (This holds no matter k, as $r^{\max}$ increases as k decreases to preserve indifference.)

Since C_t's expected payoff from entering is strictly decreasing in his belief about future counterparties' strategies $\sigma_{t'}$ (see, e.g., Eq. (B.46)), he stays out if $\sigma_{t'}$ decreases. □

B14. Proof of Proposition 8

Let's start with C_t's entry condition (Eq. (B.42)): given $\tilde{k}_t \sim \text{Pareto}(k_0, 1)$, we can replace σ by $\mathbb{P}[\sigma_t = 1] = 1 - k_0/k^*$ and given stationarity we can set $\lambda = 0$. C_t must be indifferent at the cut-off k^*:

$$\frac{k^*}{1-\eta} = \frac{\rho\big(R-r\big)}{\rho + (1-\rho)\theta\big[1-\eta\big(1-\frac{k_0}{k^*}\big)\big]}. \tag{B.65}$$

Solving for k^* gives

$$k^* = \frac{\rho(1-\eta)(R-r) - (1-\rho)\theta\eta k_0}{\rho + (1-\rho)\theta(1-\eta)}. \tag{B.66}$$

□

B15. Proof of Proposition 9

By Proposition 3, B can invest in (y', ℓ') but not in (y, ℓ) if and only if

$$\max v_{\text{b.note}}\Big|_{(y,\ell)} < c \le \max v_{\text{b.note}}\Big|_{(y',\ell')}, \tag{B.67}$$

where $\max v$ is as defined in Eq. (B.35). Substituting for R, r and σ in the value of the banknote (Eq. (17) with $\lambda = 0$ given stationarity), this says that

$$\frac{\rho y + (1-\rho)\theta(1-\eta)\ell}{\rho + (1-\rho)\theta(1-\eta)} < c \le \frac{\rho y' + (1-\rho)\theta(1-\eta)\ell'}{\rho + (1-\rho)\theta(1-\eta)}. \tag{B.68}$$

There exists c satisfying the above inequalities whenever the left-most term is less than the right-most term. This reduces to the condition in the proposition (Eq. (32)). □

B16. Proof of Proposition 10

Observe first that the value of the banknote is given by the same expression as in the baseline model (Eq. (17) with $\lambda = 0$ given stationarity). But now an interior value of σ is determined by counterparties' entry condition. Recall that the matching function is homogenous, so each counterparty is matched with a debtholder with probability σ/q. Counterparties' entry condition is thus

$$\frac{\sigma}{q}\big(v - p\big) \ge k, \tag{B.69}$$

where q represents the steady-state mass of counterparties entering at each date. Since each counterparty is small, the inequality above will bind. Substituting in for v and $p = \eta v + (1-\eta)r$ from Lemma 2, we have

$$\frac{\sigma}{q}\left(\frac{\rho\big(R-r\big)}{\rho + (1-\rho)\theta(1-\eta\sigma)}\right) = \frac{k}{1-\eta}. \tag{B.70}$$

With $\sigma = m\sqrt{q}$, this can be re-written as

$$mk(1-\rho)\theta\eta q - k\big(\rho + (1-\rho)\theta\big)\sqrt{q} + m\rho(1-\eta)(R-r) = 0. \tag{B.71}$$

This is a quadratic equation in $\sqrt{q}$. It has the two solutions, i.e. there are two steady states,

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Appendix C. Table of Notations

Players and Indices	
t	time index
B	borrower or "bank"
C_t	(potential) creditor/counterparty at Date t
H_t	debtholder at Date t
Technologies and Preferences	
y	payoff of B's investment
c	cost of B's investment
ℓ	liquidation value of B's investment
ρ	probability B's investment pays off each date
θ	probability C_t is hit by liquidity shock at each date
u	B's utility (used only in the Appendix)
k	C_t's entry cost
Prices, Values, and Strategies	
R	terminal repayment (face value of debt)
r	redemption value
v^{s_t}	value of B's debt to a creditor in state s_t
p^{s_t}	secondary-market price of B's debt in state s_t
σ^{s_t}	mixed strategy of counterparty C_t
Other Variables	
s_t	state/sunspot at Date t
λ	$\mathbb{P}[s_{t+1} = 0 \mid s_t = 1]$, "confidence crisis" probability
$\max v$	debt capacity/maximum value of an instrument (Eq. (B.35))
$r^{\max}$	maximum redemption value s.t. C_t enters (Eq. (21))
m	matching parameter in Section 6.3

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