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Warehouse banking☆

Jason Roderick Donaldson[a,b], Giorgia Piacentino[b,c], Anjan Thakor[a,d,*]

[a] Olin Business School, Washington University in St. Louis, 1 Brookings Drive, St Louis MO 63130, USA
[b] CEPR, 3 Great Sutton Street, London EC1V 0DX
[c] Columbia Business School, Columbia University, 022 Broadway, New York, NY 10027, USA
[d] ECGI, c/o the Royal Academies of Belgium, Palace of the Academies, Rue Ducale 1 Hertogsstraat, 1000 Brussels - Belgium

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ABSTRACT

We develop a theory of banking that explains why banks started out as commodities warehouses. We show that warehouses become banks because their superior storage technology allows them to enforce the repayment of loans most effectively. Further, interbank markets emerge endogenously to support this enforcement mechanism. Even though warehouses store deposits of real goods, they make loans by writing new fake warehouse receipts, rather than by taking deposits out of storage. Our theory helps to explain how modern banks create funding liquidity and why they combine warehousing (custody and deposit-taking), lending, and private money creation within the same institutions. It also casts light on a number of contemporary regulatory policies.

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> The banks in their lending business are not only not limited by their own capital; they are not, at least immediately, limited by any capital whatever; by concentrating in their hands almost all payments, they themselves create the money required.... —Wicksell (1907)

1. Introduction

Banking is an old business. Banks evolved from ancient warehouses, specifically from warehouses whose deposit receipts served as private money. The connection between banking and warehousing is fundamental. Throughout history, banks have evolved systematically from warehouses. For example, before modern banking, depositories of barley and silver in ancient Mesopotamia and grain silos in ancient Egypt started to resemble banks (Geva, 2011; Westermann, 1930); the first banks came to be in ancient Greece, where money changers, "[e]xperts at keeping their own

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* Corresponding author at: Olin Business School, Washington University in St. Louis, 1 Brookings Drive, St Louis MO 63130, USA.

E-mail address: thakor@wustl.edu (A. Thakor).

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cash secure, began safeguarding valuables for others," after which their deposit receipts "became a type of money," and they eventually took the "final step to becoming full-fledged bankers [by making] larger loans" (Roberts, 2011, p. 72). Goldsmith bankers arose in early modern Europe due to their superior safes for storing "money and plate in trust" (Richards, 1934, p. 35; Lawson, 1855),[1] and rice storage facilities began the practice of fractional reserve banking in seventeenth century Japan (Crawcour, 1961). Similarly, tobacco warehouses were instrumental in the creation of banking and payments in eighteenth century Virginia, where warehouse receipts were ultimately made legal tender (Davies, 1994). Later, in the nineteenth century, granaries were doing banking in Chicago (Williams, 1986); and even today grain silos in Brazil perform banking activities (Skrastins, 2015). However, current banking theories are not linked to this evolution of banks from warehouses. This raises the questions we address in this paper. Why did banks start out as warehouses? And what are the implications of banks' warehousing function for contemporary bank regulation?

To address these questions, we develop a theory of banking that is linked to these historical roots. It explains why banks offer deposit-taking, account-keeping, and custodial services—i.e. warehousing services—within the same institutions that provide lending services. The theory sheds new light on the importance of interbank markets and banks' private money creation. It also offers a new perspective on regulatory policies, such as capital requirements and monetary policy.

Model preview. In the model, an entrepreneur has a productive investment project. He needs to hire a worker to do the project, but his endowment is limited. Further, the output of his project is not pledgeable,[2] so he cannot pay the worker on credit. After his project pays off, the entrepreneur needs to store his output before he consumes. He can store it privately, in which case it depreciates, or he can store it in a warehouse, in which case it does not depreciate. This superior storage technology of the warehouse could reflect the fact that the warehouse prevents spoilage like grain silos in ancient Egypt or protects against theft like safes in early modern Europe.[3] Further, warehouse deposits are publicly observable, and hence pledgeable.[4]

Results preview. Our first main result is that the entrepreneur is able to borrow from the warehouse to finance his project, even though his output is nonpledgeable. The warehouse can overcome the nonpledgeability problem because the entrepreneur wants to store his deposits in the warehouse, and the warehouse has the right to seize the deposits of a defaulting borrower as repayment—banks still have this right today, called banker's setoff. Thus, warehouses' superior storage technology makes it incentive compatible for the entrepreneur to repay his debt to access warehouse storage. This mechanism explains why the same institutions should provide both the warehousing and lending services in the economy.[5]

Our second main result is that interbank markets, i.e. inter-warehouse markets, for the entrepreneur's debt are sufficient to support this enforcement mechanism even if the entrepreneur can borrow from one warehouse and store his output in another. This is because this other warehouse can buy the entrepreneur's debt in the interbank market, thereby obtaining the right to seize the entrepreneur's deposits.[6] As a result, the entrepreneur ends up repaying in full no matter which warehouse he deposits in. This finding can shed light on why successful banking systems throughout history, such as those operated by Egyptian granaries and London goldsmiths, as well as those in existence today, have indeed developed interbank networks.[7]

Our third main result is that warehouse banks make loans even if they have no initial deposits to lend out. In fact, they lend the constrained-efficient amount by making loans in new fake warehouse receipts—i.e. receipts to redeem deposits that are not backed by current deposits.[8]

[1] These goldsmiths owned safes that gave them an advantage in safekeeping. This interpretation is emphasized in He et al. (2005,2008) as well as in many historical accounts of banking, including, for example, the Encyclopedia Britannica, which states that "The direct ancestors of modern banks were the goldsmiths. At first the goldsmiths accepted deposits merely for safekeeping, but early in the seventeenth century their deposit receipts were circulating in place of money" (1954, vol. 3, p. 41). Related accounts appear in economics textbooks such as Baumol and Blinder (2009), Mankiw (2008), and Greenbaum et al. (2015).

[2] See Holmström and Tirole (2011, p. 3) for a list of "...several reasons why this (nonpledgeability) is by and large reality."

[3] Allen and Gale (1998) also assume that the storage technology available to banks is strictly more productive than the storage technology available to consumers.

[4] We relax the assumption that warehouse deposits are pledgeable in Section 4.3.

[5] Empirical evidence that seems to support this result appears in Skrastins (2015). Using a differences-in-differences research design, Skrastins (2015) shows that agricultural lenders in Brazil extend more credit when they merge with grain silos, i.e. banks lend more when they are also warehouses.

[6] This seizure right is the right to set off two mutual debts: the deposit from the entrepreneur and the loan acquired from the original warehouse in the interbank market. Such rights to set off debts arising from distinct transactions date back to Roman law (Loyd, 1916); see Section 3.3 for more on this.

[7] See Geva (2011, p. 141) for a description of how warehouse banks in Greco-Roman Egypt relied on inter-granary transfers that were entirely based on accounts. See Quinn (1997) and Geva (2011) for analyses of interbank networks that existed among London goldsmiths.

[8] We refer to these new receipts as "fake receipts" due to their lack of deposit backing, although we emphasize that they are good-value IOUs. Note that other authors have used this term before; however, they have suggested that when banks create money they are performing a kind of swindle. For example, Rothbard (2008) says "banks have habitually created warehouse receipts (originally bank notes and now deposits) out of thin air. Essentially, they are counterfeiters of fake warehouse-receipts to cash or standard money, which circulate as if they were genuine, fully backed notes or checking accounts.... This sort of swindling or counterfeiting is dignified by the term 'fractional-reserve banking.'" Our findings contrast with this perspective. For us, the creation of such private money is no swindle—it is essential for banks to extend the efficient level of credit. However, the quantity of receipts the warehouse can issue is limited by the entrepreneur's ability to repay his debt, and hence the worker's willingness to accept them to use them as a store of value. This finding connects Tobin's (1963) work to the origins of banks as warehouses (see below). Further, it appears that making loans in private money has long been an accepted part of banking; e.g., in seventeenth-century London, depositors with goldsmiths seem to have been fully aware that their deposits might be lent out, as are the depositors in our

Thus, when a warehouse bank makes a loan, it is not reallocating cash deposits into loans on the left-hand side of its balance sheet. Rather, it is creating a new liability—it is lending out fake deposit receipts, expanding its balance sheet. The entrepreneur uses these receipts to pay the worker, who accepts them to access the warehouse's superior storage technology. In this way, warehouse receipts emerge as a medium of exchange because they are a store of value. Thus, the warehouse's superior storage technology allows it not only to enforce the repayment of loans but also to create circulating private money to make loans, i.e. to make loans by creating deposits. This is reminiscent of Keynes:

> It is not unnatural to think of deposits of a bank as being created by the public through the deposits of cash representing either savings or amounts which are not for the time being required to meet expenditures. But the bulk of the deposits arise out of the action of the banks themselves, for by granting loans, allowing money to be drawn on an overdraft or purchasing securities, a bank creates a credit in its books which is the equivalent of a deposit (Macmillan Committee, 1931).

As in this description, a loan is just an exchange of IOUs in our model—the entrepreneur gives the warehouse a promise to repay (the loan) and, in exchange, the warehouse gives the entrepreneur a promise to repay (the deposit receipt). However, this seemingly zero-net transaction circumvents the entrepreneur's nonpledgeability problem and thus has a positive effect on aggregate output.[9]

Application to modern versus historical banks. Despite our focus on the origins of banking, our model illuminates some aspects of modern banking as well. Modern banks are complex institutions that perform many important functions outside of our model.[10] However, the storage and payments services that warehouse banks provide in our model remain fundamentally important for banks after hundreds of years. Instead of providing safekeeping for grain or gold and issuing receipts that serve as a means of payment, modern banks create bank accounts for storing wealth and issue claims, checkbooks, and cards that serve as means of payment. In the model, loan repayment is ensured by the threat of excluding a delinquent borrower from warehousing services. Modern banks too benefit from similar advantages for storing money and financial securities.[11] Recently, exclusion from the banking system made the high costs of storing cash salient for some firms in Colorado. Specifically, marijuana businesses have had their bank accounts closed, forcing them to store cash privately. The costs of private storage are reflected in the following quote from the New York Times: "[Marijuana entrepreneur] Dylan Donaldson... knows the hidden costs of a bank-challenged business. He has nine 1000-pound safes bolted to the floor in...his dispensary [and] he pays $100,000 a year for armed guards."[12]

Modern banks warehouse goods, such as money and financial securities, that serve as stores of value. Likewise, their immediate predecessors (e.g. London goldsmiths) warehoused goods, such as precious metals, that served as stores of value. However, warehouses in our baseline model warehouse the consumption/production good, rather than a good that serves specifically as a store of value. Thus, our baseline warehouses may seem formally closer to grain warehouses in ancient times than to modern banks (see Richards, 1934; Roberts, 2011). However, in an extension, we show that warehouses can still use their superior storage technology to enforce repayment if they specialize in warehousing a durable good such as gold, i.e. our results do not depend on the good the warehouse specializes in storing (Section 5.1). Intuitively, the reason is that if a borrower wants to save, he always holds gold. Since the warehouse has a superior technology for storing gold, he wants to deposit his gold in a warehouse. Thus, he has incentive to repay his debt to access warehouse storage.

Policy. Our model is rooted in history, but provides a perspective on contemporary bank regulation and offers support for some current policy proposals. First, we ask how bank equity affects lending. To create a role for warehouse-bank equity, we relax the assumption that warehouse deposits are perfectly pledgeable. We find that warehouse banks need equity to extend credit, because this mitigates the nonpledgeability problem between banks and depositors. This result complements the skin-in-the-game argument for bank capital that appears in Coval and Thakor (2005), Holmström and Tirole (1997), Mehran and Thakor (2011), and Rampini and Viswanathan (2015); see Thakor (2014) for a review of this literature.[13] Bank eq-

model (see Selgin, 2012). A related expansion of bank balance sheets occurs in the textbook relending model of bank money creation—the so called "money multiplier" associated with fractional-reserve banking (see, e.g., Samuelson, 1980, ch. 16). In this model, a bank takes deposits and lends them out. Then, later, the deposits are deposited back in the bank, expanding the balance sheet. That is, the money multiplier is created when the bank keeps only a fraction of its deposits on reserve, lending out the rest. By contrast, warehouse banks in our model make loans even with no deposited goods, and this expansion of economic activity occurs with a single transaction—a loan. Borrowers use warehouse receipts as working capital to make productive investments. As a result, making a loan by issuing new receipts creates an intertemporal transfer of liquidity, improving efficiency.

[9] This is consistent with Quinn and Roberds' (2014) empirical finding that the Bank of Amsterdam's ability to create unbacked private money allowed it to finance its loans and resulted in the bank florin becoming the dominant international currency throughout Europe.

[10] Important bank functions include risk sharing (Diamond and Dybvig, 1983), delegated monitoring (Diamond, 1984), and screening (Coval and Thakor, 2005; Ramakrishnan and Thakor, 1984). Historically, after warehouses began lending due to their superior storage technology, they would have had incentive to develop further banking-specific expertise in these areas.

[11] The costs of private storage of money are reflected in the negative bond yields that currently prevail in Japan, Switzerland, and around the eurozone. Further, in 2011, even before sovereign rates became negative, the Bank of New York Mellon, which is the largest depository institution in the world today, charged its depositors a fee to hold cash (see, e.g., Rappaport, 2011). This bank is usually classified as a custodian bank, i.e. an institution responsible for the safeguarding, or warehousing, of financial assets.

[12] Richtel (2015).

[13] Our result that higher bank capital leads to more liquidity creation is consistent with Berger and Bouwman's (2009) evidence for the the US banking system. But it is in contrast to liquidity-creation theories. In Bryant (1980) and Diamond and Dybvig (1983) bank capital plays no role,

uity was historically important for banks to create circulating banknotes, i.e. receipts. Indeed, some banknotes in the Free Banking Era were embossed with the amount of equity held by the issuing bank.

We also analyze a reduced form of monetary policy. We model the policy rate as the return on warehouse banks' assets, since it is the rate of return on bank reserves in reality. We find that increasing the policy rate can *increase* the supply of credit—"tighter" monetary policy "loosens" credit in our model. This is because when warehouse banks can store at a higher rate, borrowers have a stronger incentive to repay their loans in order to access this high savings rate. This mitigates the effects of nonpledgeability ex post, leading to more credit ex ante.

Our model addresses these policies entirely through the lens of banks' warehousing function. However, abstracting from banks' other functions is not without some cost. For example, we make the assumption that bank loans are riskless, which prevents us from speaking to a number of contemporary policy-relevant issues, such as risk-based capital regulation and insolvency-risk-induced runs.[14]

Related literature. We make four main contributions relative to the literature.

First, we point out two distinguishing features of warehouses that make them the natural banks: (i) they prevent depreciation or theft so that goods stored in a warehouse earn a better rate of return than goods stored privately, and (ii) goods stored in a warehouse are pledgeable, unlike the entrepreneur's output. This second feature is essential for deposit-taking, as Gu et al. (2013) emphasize. Our contribution relative to this paper is to show that the storage technology gives warehouses an advantage not only in taking deposit, but also in enforcing loans. Thus, our model provides a new rationale for why deposit-taking and lending should be done in the same institution. This complements the analysis in Kashyap et al. (2002) who argue that these two functions of a bank should go together since combining them enables the bank to hold cash as insurance against both withdrawals of demand deposits and takedowns of credit lines (loans). We abstract from this liquidity insurance channel, since we focus on banks' creating private money (warehouse receipts), which could allow them to meet drawdowns by issuing new receipts.

Second, we show that the secondary market for defaulted debt prevents a borrower from avoiding repayment by depositing his output in a warehouse different from the one he initially borrowed from. This is related to Broner et al.'s (2010) finding that secondary markets for sovereign debt reduce strategic default. In that model, a sovereign defaults if its debt is held by foreign investors. However, these foreign investors are still willing to lend to the sovereign because they anticipate being able to sell their debt to domestic investors. We add to this result in three ways: (i) we analyze how secondary markets deter borrowers from diverting output and storing "abroad" in "foreign" warehouses; (ii) we show that secondary markets support the private enforcement of repayments via seizure; and (iii) we point out the special importance of interbank markets: banks have the ability to enforce debts via seizure because they hold borrowers' deposits in storage.

Third, in our model, banks must lend in fake receipts to extend the efficient level of credit. Even though these receipts are not backed by real deposits, the worker accepts them to access warehouses' superior storage technology. Thus, in our model, bank money creation expands the supply of credit as in the verbal descriptions of Hahn (1920) and Wicksell (1907) and the reduced-form models of Bianchi and Bigio (2015) and Jakab and Kumhof (2015).[15] However, our model is consistent with Tobin's (1963) critique that banks cannot create money beyond the demand for savings, i.e. storage in warehouse-bank deposits. But, contrary to Tobin's view, bank money creation can itself increase the equilibrium amount of deposits in our model, since it increases aggregate output by mitigating the nonpledgeability friction. However, this is not a "widow's curse" in which banks can create an arbitrary amount of money, since money creation is limited by the entrepreneur's ability to repay his debt and the worker's willingness to save in fake receipts.

Fourth, warehouses are effectively able to enforce exclusion from financial markets, which allow for efficient savings/storage in our model. We show that they can implement exclusion in a finite horizon model, even if they are not able to make long-term commitments. In other words, exclusion from storage at the final date is subgame perfect. This adds to the results in Bolton and Scharfstein (1990), who show that the threat of exclusion from credit markets can mitigate incentive problems in corporate finance with commitment, and Bulow and Rogoff (1989), who analyze how the threat of exclusion can mitigate incentive problems in sovereign debt markets with an infinite horizon.[16]

Our paper is also related to the literature on banking theory and liquidity creation, including Allen and Gale (1998,2004), Allen et al. (2014), Bryant (1980), Diamond (1984), Diamond and Dybvig (1983),

whereas in DeAngelo and Stulz (2015) and Diamond and Rajan (2001), higher bank capital leads to lower liquidity creation.

[14] See, however, Calomiris et al. (2015) for a model that does speak to these issues. In that model, cash reserve requirements improve banks' risk management incentives as they pertain to their (non-cash) risky assets. Liquidity management thus has solvency ramifications, an issue beyond our scope.

[15] Our paper is also related to papers in which debt serves as inside money generally. For example, Kahn and Roberds (2007) develop a model that shows the advantage of circulating liabilities (transferable debt) over simple chains of credit. Townsend and Wallace (1987) develop a model of pure intertemporal exchange with informationally separated markets to explain the role of circulating liabilities in exchange. We also provide a framework to study private money creation in a relatively classical model (a Walrasian equilibrium subject to appropriate constraints). Brunnermeier and Sannikov (2016), Kiyotaki and Moore (2002), and Hart and Zingales (2015) provide complementary Walrasian models in which bank money creation is valuable because it creates safe and/or resaleable liabilities. Within such a model, Wang (2016) examines unconventional monetary policy and its effect on bank money creation.

[16] Bond and Krishnamurthy (2004) study the problem of credit market exclusion with competing banks. They show that the constrained-efficient outcomes can be achieved if default by a borrower allows the lender to seize funds that the borrower has deposited with other banks. In out model, the interbank market facilitates the enforcement of repayment so that loan sales between banks deliver the same outcome.

Donaldson and Piacentino (2017), Millon and Thakor (1985), Postlewaite and Vives (1987), and Ramakrishnan and Thakor (1984).

Layout. In Section 2, we describe the environment and present two benchmarks: the first-best allocation and the allocation with no credit. In Section 3, we characterize the equilibrium. In Section 4, we study liquidity creation and policy implications. In Section 5, we show that our conclusions are robust to the inclusion of a second good which serves as a store of value and to different utility specifications. In Section 6, we conclude. The appendix contains all proofs and a glossary of notations.

2. Environment and benchmarks

In this section, we present the environment and two benchmark allocations. Given our historical motivation, we frame the model in terms of farmers who hire laborers to plant grain. Farmers want to deposit output in warehouses, i.e. grain silos, which prevent grain from depreciating.

2.1. Timeline, production technology, and warehouses

There are three dates—Date 0, Date 1, and Date 2—and three groups of players—farmers, warehouses, and laborers/workers. There is a unit continuum of each type of player. There is one real good, called grain, which serves as the numeraire.[17] There are also receipts issued by warehouses, which entail the right to withdraw grain from a warehouse.

All players are risk neutral and consume only at Date 2.[18] Denote farmers' consumption by c^f, laborers' consumption by c^l, and warehouses' consumption by c^b (the index b stands for bank). Farmers have an endowment e of grain at Date 0. No other player has a grain endowment. Laborers have labor at Date 0. They can provide labor ℓ at the constant marginal cost of one. So their utility is $c^l - \ell$. Farmers have access to the following technology. At Date 0, a farmer invests i units of grain and ℓ units of labor. At Date 1, this investment yields

$$y = A \min\{\alpha i, \ell\}, \tag{1}$$

i.e. the production function is Leontief.[19]

We make two main assumptions on technologies. First, farmers' output y is not pledgeable and, second, warehouses have a superior storage technology. Specifically, if grain is stored privately, it depreciates at rate $\delta \in (0, 1)$: if player j stores s_t^j units of grain privately from Date t to Date $t+1$, he has $(1-\delta)s_t^j$ units of grain at Date $t+1$. In contrast, if grain is stored in a warehouse, it does not depreciate.[20]

Note that only the output of the farmers' technology is not pledgeable.[21] Warehoused grain is pledgeable and, as a result, warehouses can issue receipts as "proof" of their deposits. These receipts are enforceable against the issuing warehouse and payable to the "bearer upon demand," so the bearers of receipts can trade them among themselves. Warehouses can issue these "proof-of-deposits" receipts even when there is no deposit. These receipts, which we refer to as "fake receipts," still entail the right to withdraw grain from a warehouse, and thus they are warehouses' liabilities that are not backed by the grain they hold.

2.2. Parameter restrictions

We now impose two restrictions on the deep parameters of the model. The first ensures that farmers' production technology generates sufficiently high output that the investment has positive NPV in equilibrium. The second ensures that the incentive problem that results from the nonpledgeablity of farmers' output is sufficiently severe to generate a binding borrowing constraint in equilibrium. Note that since the model is linear, if a farmer's borrowing constraint does not bind, he will scale his production infinitely.

Parameter Restriction 1. The farmers' technology is sufficiently productive,

$$A > 1 + \frac{1}{\alpha}. \tag{2}$$

Parameter Restriction 2. Depreciation from private storage is not too high,

$$\delta A < 1. \tag{3}$$

2.3. Benchmark: first best

We now consider the first-best allocation, i.e. the allocation that maximizes utilitarian welfare subject only to the aggregate resource constraint. Since the utility, cost, and production functions are all linear, in the first-best allocation all resources are allocated to the most productive players at each date. At Date 0 the farmers are the most productive and at Date 1 the warehouses are the most productive. Thus, all grain is held by farmers at Date 0 and by warehouses at Date 1. Laborers exert labor in proportion $1/\alpha$ of the total grain invested to maximize production.

Proposition 1. (First-best allocation) The first-best labor and investment allocations are given by $\ell_{\text{fb}} = \alpha e$ and $i_{\text{fb}} = e$. All grain is stored in warehouses at Date 1.

2.4. Benchmark: no credit

Consider a benchmark model in which there is no lending. Recall that only farmers have an endowment at Date

[17] In Section 5.1, we introduce a second good, "gold," to discuss the evolution of banks from warehouses of precious metals and other valuables.

[18] We relax these assumptions in Section 5.2, where we consider farmers with logarithmic utility over consumption at Date 1 and Date 2.

[19] None of our main results depends on the functional form of the production function. For example, if labor were the only input to the farmers' production function all the results would go through.

[20] Historically, warehouses had a physical advantage in storing goods or safeguarding valuables. And, even today, physical safes play an important role in banking, even in developed economies. For example, custodians like Clearstream hold physical certificates for publicly listed companies in Germany.

[21] We partially relax the assumption that warehouse deposits are pledgeable in Section 4.3.

0; warehouses and laborers have no endowments. Thus, the benchmark with no lending is tantamount to a model in which warehouses cannot lend in fake receipts. Here, farmers simply divide their endowment between their capital investment i and their labor investment ℓ; their budget constraint reads

$$i + w\ell = e, \tag{4}$$

where w is the wage paid to laborers. The Leontief production function implies that they will always make capital investments equal to the fraction α of their labor investments, or

$$\alpha i = \ell. \tag{5}$$

We summarize the solution to this benchmark model in Proposition 2 below.

Proposition 2. (Benchmark with no credit) With no credit, the equilibrium is as follows:

$$\ell_{\text{nc}} = \frac{\alpha e}{1+\alpha}, \tag{6}$$

$$i_{\text{nc}} = \frac{e}{1+\alpha}. \tag{7}$$

Note that even though warehouses do not improve efficiency by extending credit to farmers, they still provide a useful service in the economy by taking grain deposits and providing efficient storage of grain from Date 1 to Date 2. Below we will see that farmers' incentive to access this storage technology is what makes them repay their debt, making the warehouses not only deposit-takers but also lenders, and thus making them banks.

3. Equilibrium with incentive constraints

In this section, we consider the equilibrium of the model in which farmers' repayments must be incentive compatible.

3.1. Financial contracts

There are three types of contracts in the economy: labor contracts, deposit contracts, and lending contracts. We restrict attention to bilateral contracts, although liabilities are tradeable: farmers use warehouse receipts to pay laborers and warehouses trade farmers' debt in an interbank market.

Labor contracts are between farmers and laborers. Farmers pay laborers $w\ell$ in exchange for laborers' investing ℓ in the production technology y.

Deposit contracts are between warehouses and the other players, i.e., laborers, farmers, and (potentially) other warehouses. Warehouses accept grain deposits with gross rate R_t^D over one period, i.e. if player j makes a deposit of d_t^j units of grain at Date t, he has the right to withdraw $R_t^D d_t^j$ units of grain at Date $t+1$. When a warehouse accepts a deposit of one unit of grain, it issues a receipt in exchange as "proof" of the deposit.

Lending contracts are between warehouses and farmers. Warehouses lend L to farmers at Date 0 in exchange for farmers' promise to repay $R^L L$ at Date 1, where R^L is the lending rate. Warehouses can trade farmers' debt at Date 1 in the interbank market.

When warehouses make loans, they can either lend grain or issue new receipts. A loan made in receipts is tantamount to a warehouse offering a farmer a deposit at Date 0 in exchange for the farmer's promise to repay grain at Date 1. When a warehouse makes a loan in receipts, we say that it is "issuing fake receipts." We refer to a warehouse's total deposits at Date t as D_t. These deposits include both those deposits backed by grain and those granted as fake receipts.

The timeline of moves for each player and their contractual relationships are illustrated in Fig. 1.

3.2. Incentive constraint

Lending contracts are subject to a form of limited commitment on the farmers' side. Because farmers' Date 1 output is not pledgeable, they are free to divert their output and store it privately. If they deposit their output in a warehouse, the warehouse can seize the grain that the farmer owes it. The next proposition gives a condition for the farmer to prefer to deposit in a warehouse and repay his debt than to store privately.

Proposition 3. (Incentive constraint) If a farmer has grain g and debt $R^L B$ at Date 1, he prefers to repay his debt than to store privately if and only if

$$R_1^D\left(g - R^L B\right) \geq (1-\delta)g. \tag{IC}$$

The incentive constraint above says that to repay his debt, the return R_1^D on warehouse deposits must be sufficiently high relative to the return $1-\delta$ on private storage.

3.3. Interbank market

The next result says that the incentive constraint in Proposition 3 is not only necessary but also sufficient for a farmer to repay his debt. This is a result of the fact that warehouses can trade farmers' debt in the interbank market.

Proposition 4. (Interbank markets enforce repayment) Given the interbank market for farmers' debt, a farmer cannot avoid repayment by depositing his output in a warehouse different from the warehouse he originally borrowed from. A farmer's global incentive constraint is as in Proposition 3.

To see the mechanism behind this result, consider the case in which there are two warehouses, called Warehouse 0 and Warehouse 1. Suppose that a farmer borrows from Warehouse 0 at Date 0, but deposits his output with Warehouse 1 at Date 1. Now, given Warehouse 1 holds the farmer's deposit, it has a superior ability to enforce repayment. Thus, it buys the loan from Warehouse 0 in the interbank market.[22] Now Warehouse 1 holds both the

[22] In the proof of the proposition, we show that farmers' debt trades in the market at par, since it is effectively a riskless bond. As a result, a lending warehouse does not risk selling the farmers' debt at a discount. This means that the repayment the original warehouse receives is inde-

Date 0	Date 1	Date 2
Warehouses accept deposits D_0 lend L to farmers store s_0^b	Warehouses accept deposits D_1 repay $R_0^D D_0$ to depositors store s_1^b trade in interbank market	Warehouses repay $R_1^D D_1$ to depositors consume $c^b = s_1^b - R_1^D D_1$
Farmers borrow B from warehouses invest i and ℓ in technology y pay laborers $w\ell$ deposit d_0^f in warehouses store s_0^f	Farmers receive cash flow $y(i, \ell)$ receive $R_0^D d_0^f$ from warehouses have total grain holding g_1^f deposit d_1^f in warehouses store s_1^f	Farmers receive $R_1^D d_1^f$ from warehouses consume $c^f = R_1^D d_1^f + (1-\delta)s_1^f$
Laborers exert labor ℓ accept wage $w\ell$ deposit d_0^l in warehouses store s_0^l	Laborers receive $R_0^D d_0^l$ from warehouses deposit d_1^l in warehouses store s_1^l	Laborers receive $R_1^D d_1^l$ from warehouses consume $c^l = R_1^D d_1^l + (1-\delta)s_1^l$

Fig. 1. Timeline.

farmer's debt and his deposit, so if the farmer defaults, the warehouse can just seize his deposit. In summary, the farmer's repayment does not depend on the warehouse in which he deposits.

Observe that after trading in the interbank market, the farmer and the warehouse have bilaterally reciprocal debts—the deposit is a debt from the warehouse to the farmer, and the loan is a reciprocal debt from the farmer to the warehouse. Hence, the seizure of deposits can also be interpreted as the setoff of these reciprocal debts. Our analysis therefore implies that the right of bilateral setoff plus a market for loans creates a multilateral enforcement mechanism. Notably, our mechanism does not rely on multilateral or "triangular" setoff, although this could make things work more smoothly in practice, suggesting a rationale for the master agreements that facilitate multilateral netting in interbank markets (Anderson et al., 2009; Corbi, 2012).[23]

We have now established that a farmer repays in full, as long as his debt is not too high relative to his output. In other words, even though his output is not pledgeable by assumption, a fraction of it is "effectively pledgeable," since he has incentive to make the repayment in order to access a warehouse's superior storage technology.

3.4. Individual maximization problems and equilibrium definition

We now turn to the definition of the market equilibrium. All players take prices as given and maximize their Date 2 consumption subject to their budget constraints. Farmers' maximization problems are also subject to their incentive compatibility constraint (IC).

The warehouse's maximization problem is

$$\text{maximize} \quad c^b = s_1^b - R_1^D D_1 \tag{8}$$

over $s_1^b, s_0^b, D_0, D_1,$ and L subject to

$$s_1^b = R^L L + s_0^b - R_0^D D_0 + D_1, \tag{BC_1^b}$$

$$s_0^b + L = D_0, \tag{BC_0^b}$$

and the nonnegativity constraints $D_t \geq 0, s_t^b \geq 0,$ and $L \geq 0$. To understand this maximization program, note that Eq. (8) says that the warehouse maximizes its consumption c^b, which consists of the difference between what is stored in the warehouse at Date 1, s_1^b, and what is paid to depositors, $R_1 D_1$. Eq. (BC_1^b) is the warehouse's budget constraint at Date 1, which says that what is stored in the warehouse at Date 1, s_1^b, is given by the sum of the interest on the loan to the farmer, $R^L L$, the warehouse's savings at Date 0, and the deposits at Date 1, D_1, minus the interest the warehouse must pay on its time 0 deposits, $R_0^D D_0$. Similarly, Eq. (BC_0^b) is the warehouse's budget constraint at Date 0, which says that the sum of the warehouse's savings at Date 0, s_0^b, and its loans, L, must equal the sum of the Date 0 deposits, D_0.

pendent of the warehouse the farmer deposits in—it is repaid in full as long as it is incentive compatible for the farmer to deposit.

[23] Our model thus applies to legal environments in which counterparties can set off bilateral debts, even if they arise from different transactions. This is the case today—triangular setoff rights are limited, but bilateral setoff rights are not (Rosenbaum et al., 2009). It has also been the case for much of history. But not for all of history, since the law sometimes limited the right to set off bilateral debts arising from different transactions (Loyd, 1916). That said, we would like to suggest another interpretation of setoff in our model. Suppose a borrower defaults and his bank (i.e. his warehouse) simply does not honor his deposits. The borrower knows that if he shows up in court, the bank will demand repayment of the loan, with the possibility of debtors' prison for the defaulting borrower. Thus, the borrower will be hard pressed to take the bank to court to enforce his right to the deposit. This means that a defaulting borrower is effectively excluded from the banking system. This is the core of the enforcement mechanism in our model.

The farmer's maximization problem is

$$\text{maximize} \quad c^f = R_1^D d_1^f + (1-\delta)s_1^f \tag{9}$$

over $s_1^f, s_0^f, d_0^f, i, \ell^f$, and B subject to

$$\left(R_1^D - 1 + \delta\right)\left(y\left(i, \ell^f\right) + R_0^D d_0^f + (1-\delta)s_0^f\right) \geq R_1^D R^L B, \tag{IC}$$

$$d_1^f + s_1^f - R^L B = y\left(i, \ell^f\right) + R_0^D d_0^f + (1-\delta)s_0^f, \tag{BC$_1^f$}$$

$$d_0^f + s_0^f + i + w\ell^f = e + B, \tag{BC$_0^f$}$$

and the nonnegativity constraints $s_t^f \geq 0, d_t^f \geq 0, B \geq 0, i \geq 0$, and $\ell^f \geq 0$. The farmer's maximization program can be understood as follows. In Eq. (9) the farmer maximizes his Date 2 consumption, c^f, which consists of his Date 1 deposits gross of interest, $R_1^D d_1^f$, and his depreciated private savings, $(1-\delta)s_1^f$. Eq. (IC) is the incentive compatibility constraint. Although the incentive compatibility constraint looks different from the expression in Eq. (IC) in Section 3.1, it follows directly from substitution: the farmer's Date 1 grain holding, g, comprises his Date 1 output, y, his Date 0 deposits gross of interest, $R_0^D d_0^f$, and his depreciated savings, $(1-\delta)s_0^f$. Eq. (BC$_1^f$) is the farmer's budget constraint at Date 1, which says that the sum of his Date 1 deposits and his Date 1 savings, s_1^f, minus his repayment, $R^L B$, must equal the sum of his output, y, his Date 0 deposits gross of interest, $R_0^D d_0^f$, and his depreciated savings, $(1-\delta)s_0^f$. Eq. (BC$_0^f$) is the farmer's budget constraint at Date 0, which says that the sum of his Date 0 deposits, d_0^f, his Date 0 savings, s_0^f, his investment in grain, i, and his investment in labor, $w\ell^f$, must equal the sum of his initial endowment, e, and the amount he borrows, B.

The laborer's maximization problem is

$$\text{maximize} \quad c^l = R_1^D d_1^l + (1-\delta)s_1^l - \ell^l \tag{10}$$

over $s_1^l, s_0^l, d_1^l, d_0^l$, and ℓ^l subject to

$$d_1^l + s_1^l = R_0^D d_0^l + (1-\delta)s_0^l, \tag{BC$_1^l$}$$

$$d_0^l + s_0^l = w\ell^l, \tag{BC$_0^l$}$$

and the nonnegativity constraints $s_t^l \geq 0, d_t^l \geq 0$, and $\ell^l \geq 0$. The laborer's maximization program can be understood as follows. In Eq. (10), the laborer maximizes his Date 2 consumption, c^l, which consists of his Date 1 deposits gross of interest, $R_1^D d_1^l$, and his depreciated private savings, $(1-\delta)s_1^l$, minus his cost of labor, ℓ^l. Eq. (BC$_1^l$) is the laborer's budget constraint that says that the sum of his Date 1 savings, s_1^l and his Date 1 deposits, d_1^l, must equal the sum of his Date 0 deposits gross of interest, $R_0^D d_0^l$, and his depreciated savings, $(1-\delta)s_0^l$. Eq. (BC$_0^l$) is the laborer's budget constraint at Date 0, which says that the sum of his Date 0 deposits, d_0^l, and his Date 0 savings, s_0^l, must equal his labor income, $w\ell^l$.

The equilibrium is a profile of prices $\langle R_t^D, R^L, w\rangle$ for $t \in \{0,1\}$ and a profile of allocations $\langle s_t^j, d_t^f, d_t^l, D_t, L, B, \ell^l, \ell^f\rangle$ for $t \in \{0,1\}$ and $j \in \{b, f, l\}$ that solves the warehouses' problem, the farmers' problem, and the laborers' problem defined above and satisfies the market clearing conditions for the labor market, the lending market, the grain market, and deposit market at each date:

$$\ell^f = \ell^l, \tag{MC$^\ell$}$$

$$B = L, \tag{MC^L}$$

$$i + s_0^f + s_0^l + s_0^b = e, \tag{MC$_0^g$}$$

$$s_1^f + s_1^l + s_1^b = (1-\delta)s_0^f + (1-\delta)s_0^l + s_0^b + y, \tag{MC$_1^g$}$$

$$D_0 = d_0^f + d_0^l, \tag{MC$_0^D$}$$

$$D_1 = d_1^f + d_1^l. \tag{MC$_1^D$}$$

3.5. Preliminary results for the equilibrium

Here we state three results to characterize the prices, namely the two deposit rates, R_0^D and R_1^D, the lending rate, R^L, and the wage, w. We then show that, given the equilibrium prices, farmers and laborers will never store grain privately. The results all follow from the definition of competitive equilibrium with risk-neutral agents.

The first two results say that the risk-free rate in the economy is one. This is natural, since the warehouses have a scalable storage technology with return one.

Lemma 3.1 (Deposit rates at. $t = 0$ *and* $t = 1$*)* $R_0^D = R_1^D = 1$.

Now we turn to the lending rate. Since warehouses are competitive and the farmers' incentive compatibility constraint ensures that loans are riskless, warehouses also lend to farmers at rate one.

Lemma 3.2. (Lending rate) $R^L = 1$.

Finally, since laborers have a constant marginal cost of labor, the equilibrium wage must be equal to this cost; this says that $w = 1$.[24]

Lemma 3.3. (Wages) $w = 1$.

These results establish that the risk-free rate offered by warehouses exceeds the rate of return from private storage, or $R_0^D = R_1^D = 1 > 1 - \delta$. Thus, farmers and laborers do not wish to make use of their private storage technologies. The only time a player could choose to store grain outside a warehouse is if a farmer diverts his output. However, the farmer's incentive compatibility constraint ensures he will not do this.

Corollary 1. (Grain storage) Farmers and laborers do not store grain privately, i.e., $s_0^l = s_0^f = s_1^l = s_1^f = 0$.

[24] Note that we have omitted the effect of discounting in the preceding argument—laborers work at Date 0 and consume at Date 2; discounting is safely ignored, though, since the laborers have access to a riskless storage technology with return one via the warehouses, as established above.

3.6. Equilibrium characterization

Now we characterize the equilibrium of the model. We begin by showing that, given the equilibrium prices established in Section 3.5 above, the solution to the farmers' maximization problem is a solution to the model.

Lemma 3.4. (Equilibrium program) The equilibrium allocation solves the program to

$$\text{maximize} \quad d_1^f \tag{11}$$

subject to

$$\delta\left(y\left(i, \ell^f\right) + d_0^f\right) \geq B, \tag{IC}$$

$$d_1^f + B = y\left(i, \ell^f\right) + d_0^f, \tag{BC$_1^f$}$$

$$d_0^f + i + \ell^f = e + B, \tag{BC$_0^f$}$$

and $i \geq 0, \ell^f \geq 0, B \geq 0, d_0^f \geq 0,$ *and* $d_1^f \geq 0.$

Solving the program above allows us to characterize the equilibrium allocations.

Proposition 5. (Equilibrium values of debt, labor, and investment) The equilibrium allocation is as follows:

$$B = \frac{\delta A \alpha e}{1 + \alpha\left(1 - \delta A\right)}, \tag{12}$$

$$\ell = \frac{\alpha e}{1 + \alpha\left(1 - \delta A\right)}, \tag{13}$$

$$i = \frac{e}{1 + \alpha\left(1 - \delta A\right)}. \tag{14}$$

The equilibrium above is the solution of a system of linear equations, from the binding budget constraints and the farmers' binding incentive constraints.

3.7. The equilibrium is constrained efficient (second best)

We now show that the equilibrium in our model is constrained efficient in the sense that it maximizes welfare among all individually rational incentive-compatible allocations.

Proposition 6. (The equilibrium is constrained efficient) If the worst feasible punishment for farmers is autarky, then the equilibrium summarized in Proposition 5 is optimal in the sense that it maximizes output and utilitarian welfare among all incentive-feasible allocations.

The efficiency of the equilibrium in our model suggests a rationale for the development of banks from warehouses. Exclusion from warehouse storage implements the worst feasible default penalty in our model since exclusion from storage is effectively autarky. The interbank market allows warehouse banks to implement this punishment even in our finite-horizon setting.

4. Liquidity creation, welfare, and policy

In this section, we present the analysis of the equilibrium in the context of liquidity creation. We then consider the implications of two policies: equity capital for banks and monetary policy.

4.1. Liquidity creation

We now turn to the funding liquidity warehouses create. We begin with the definition of a liquidity multiplier, which describes the total investment (grain investment plus labor investment) that farmers can undertake at Date 0 relative to the total endowment e.

Definition 1. The liquidity multiplier Λ is the ratio of the equilibrium investment in production $i + w\ell$ to the total grain endowment in the economy e,

$$\Lambda := \frac{i + w\ell}{e}. \tag{15}$$

The liquidity multiplier Λ reflects farmers' total investment at Date 0.

Proposition 7. (Fake receipts and liquidity creation) If warehouses cannot issue fake receipts, no liquidity is created, $\Lambda_{\mathrm{nr}} = 1$*. With fake receipts, the equilibrium liquidity multiplier is*

$$\Lambda = \frac{1 + \alpha}{1 + \alpha\left(1 - \delta A\right)} > 1. \tag{16}$$

Recall that warehouses have no initial endowment. Thus, if warehouses cannot issue fake receipts, they cannot lend at all. Indeed, the allocation with no fake receipts coincides with the benchmark allocation with no credit whatsoever (Proposition 2). Thus, this result implies that it is warehouses' ability to make loans in fake receipts, not their ability to take deposits, that creates liquidity. Warehouses lubricate the economy because they lend in fake receipts rather than in grain. They can do this because of their dual function: they keep accounts (i.e. warehouse grain) and also make loans. This is the crux of farmers' incentive constraints—because warehouses provide valuable warehousing services, farmers go to these warehouse banks and deposit their grain, which is then also the reason that they repay their debts.

We now analyze the effect of the private storage technology, i.e. the depreciation rate δ, on warehouses' liquidity creation. Differentiating the liquidity multiplier Λ with respect to δ,

$$\frac{\partial \Lambda}{\partial \delta} = \frac{\alpha A}{\left(1 + \alpha\left(1 - \delta A\right)\right)^2} > 0. \tag{17}$$

This leads to our next result.

Corollary 2. (Warehouse efficiency and liquidity creation) The more efficiently warehouses can store grain relative to farmers (the higher is δ), the more liquidity warehouses create by issuing fake receipts.

This result says that a decrease in the efficiency of private storage leads to an increase in overall efficiency. The reason is that δ measures the storage advantage that a warehouse has over private storage, so a higher δ weakens farmers' incentive to divert capital, thereby allowing banks to create more liquidity. We return to this result when we discuss monetary policy below.

4.2. Fractional reserves

We now proceed to analyze warehouses' balance sheets. Do warehouses actually store grain or do they lend everything out?

Proposition 8. (Deposit reserves held by warehouses) Warehouses hold a positive fraction of grain at $t = 0$; in equilibrium,

$$s_0^b = e - i = \frac{\alpha\left(1-\delta A\right)e}{1+\alpha\left(1-\delta A\right)} > 0. \tag{18}$$

Farmers have a constant-returns-to-scale technology and, therefore, in the first-best case they prefer to invest all grain in the economy in their technology, leaving no grain for storage (Proposition 1). However, warehouses still store grain in equilibrium. This is because farmers' incentive constraints put an endogenous limit on the amount that each farmer can borrow. Farmers cannot borrow enough from warehouses to pay laborers entirely in fake receipts as they would in the first best. Rather, they pay laborers in a combination of fake receipts and real grain. Laborers deposit the grain they receive in warehouses for storage.

This result implies that, although warehouses create real economic value by printing fake receipts (Proposition 7), they do not lend more than farmers are willing to repay, and they still store grain due to laborers' need to save. This squares our finding that private money is valuable with Tobin's (1963) argument that the amount banks lend and borrow must be determined to equate the supply of savings and demand for borrowing in equilibrium. That said, however, our model of risk-free lending may be too stark to speak to why contemporary banks voluntarily hold reserves, as, for example, in the solvency-cum-liquidity-risk model of Calomiris et al. (2015).

4.3. Bank capital and liquidity creation

We now extend the model to include a role for warehouse-bank capital.[25] To do this, we assume that the warehouse has equity endowment e^b at Date 1, and we add a pledgeablity problem for the warehouse. Specifically, after a warehouse accepts deposits, it has the following choice: it can either divert grain and store it privately or not divert and store the grain in the warehouse. If it diverts the grain, the depositors will not be able to claim it, but it will depreciate at rate δ.[26] If the warehouse does not divert, depositors will be able to claim it, but it will not depreciate. We show that warehouse equity has an important function—it gives the warehouse the incentive not to divert deposits.[27]

The results of this subsection follow from the analysis of the warehouses' incentive constraint, which is that depositors store in a warehouse at Date 1 only if the warehouse prefers not to divert deposits. Its payoff, if it diverts, is the depreciated value of its equity plus its deposits, or $(1-\delta)(e^b + D_1)$. Its payoff, if it does not divert, is the value of its equity plus its deposits less its repayment to its depositors, or $e^b + D_1 - R_1^D D_1$. Since $R_1^D = 1$ by Lemma 3.1, a warehouse's incentive compatibility constraint at Date 1 is

$$(1-\delta)(e^b + D_1) \le e^b. \tag{19}$$

Thus, the equilibrium allocation is constrained efficient, as summarized in Proposition 6, only if the incentive constraint above is satisfied, or

$$\frac{e^b}{D_1} \ge \frac{1-\delta}{\delta}. \tag{20}$$

This constraint says that the second best is attained only if the warehouse's capital ratio is sufficiently high. Substituting the equilibrium value of D_1 from Proposition 5 gives the next result.

Proposition 9. (Role of warehouse equity) The second-best allocation in Proposition 5 is attained only if warehouse equity is sufficiently high, or

$$e^b \ge \hat{e}^b := \frac{1-\delta}{\delta}\frac{\alpha\left[1+(1-\delta)A\right]}{1+\alpha(1-\delta A)}e. \tag{21}$$

If warehouse equity is below $\hat{e}^b$, the warehouse's incentive constraint binds (and the farmer's incentive constraint does not), and an increase in warehouse equity loosens the warehouse's incentive constraint. This allows it to accept more deposits. Since accepting more deposits allows the warehouse to obtain a larger repayment from borrowers, this also allows the warehouse to make more loans, which leads to the next result.

Proposition 10. (Liquidity creation for different levels of warehouse capital) When warehouse equity e^b is below a threshold,

$$\hat{\hat{e}}^b := \frac{\alpha(1-\delta)(1+A)e}{(1+\alpha)\delta}, \tag{22}$$

there is no lending and hence no liquidity creation. For $e^b \in (\hat{\hat{e}}^b, \hat{e}^b]$, liquidity creation is strictly increasing in warehouse

[25] Because we do not have bank failures and crises in our baseline model, our analysis likely understates the value and role of bank capital. Berger and Bouwman (2013) show that higher-capital banks have an advantage during financial crises. Calomiris and Nissim (2014) show that the market is attaching a higher value to bank capital after the 2007–2009 crisis.

[26] Note that if the warehouse diverts, it must do so at Date 1 before depositing grain in the warehouse. If it deposits grain in the warehouse from Date 1 to Date 2, it is too late to divert, because warehoused grain is publicly observable and therefore pledgeable.

[27] See Holmström and Tirole (1997) and Rampini and Viswanathan (2015) for other models in which bank capital affects incentive constraints between depositors and banks, as well as incentive constraints between banks and borrowers.

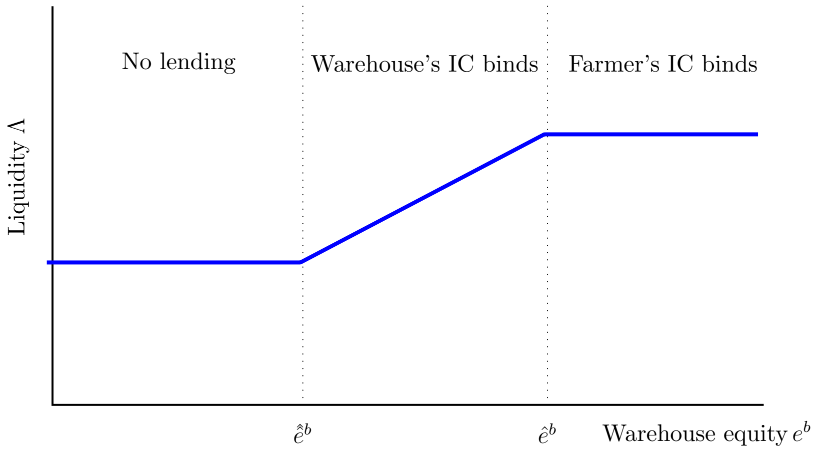

Fig. 2. The liquidity multiplier as a function of warehouse equity e^b.

equity e^b. For $e^b > \hat{e}^b$, warehouse equity has no effect on liquidity creation. Specifically,

$$\Lambda = \begin{cases} 1 \quad \text{if} \quad e^b \le \hat{\hat{e}}^b, \\ \dfrac{1+\alpha}{\alpha A - 1}\left(\dfrac{\delta}{1-\delta}\dfrac{e^b}{e} - 1\right) \quad \text{if} \quad e^b \in (\hat{\hat{e}}^b, \hat{e}^b], \\ \dfrac{1+\alpha}{1+\alpha\big(1-\delta A\big)} \quad \text{if} \quad e^b > \hat{e}^b. \end{cases} \tag{23}$$

The expression for Λ in this proposition, illustrated in Fig. 2, says that when warehouse equity is very low, the incentive problem is so severe that warehouses do not lend at all. As equity increases, warehouses start lending, and the amount they lend increases linearly until the farmers' incentive constraints bind. Above this threshold, an increase in equity has no further effect, because the farmers' incentive constraints bind.

4.4. Monetary policy

We now extend the model to analyze how monetary policy affects liquidity creation. We define the central bank rate R^{CB} as the (gross) rate at which warehouses can deposit with the central bank.[28] This is analogous to the storage technology of the warehouse yielding return R^{CB}. In this interpretation of the model, grain is central bank money and warehouse receipts are private money.

We first state the necessary analogs of the parameter restrictions in Section 2.2. Note that they coincide with Parameter Restriction 1 and Parameter Restriction 2 when $R^{\text{CB}} = 1$, as in the baseline model.

Parameter Restriction $1'$. The farmers' technology is sufficiently productive,

$$A > \frac{1}{R^{\text{CB}}} + \frac{R^{\text{CB}}}{\alpha}. \tag{24}$$

Parameter Restriction $2'$. Depreciation from private storage is not too fast,

$$A\big(R^{\text{CB}} - 1 + \delta\big) < 1. \tag{25}$$

The preliminary results of Section 3.5 lead to the natural modifications of the prices. In particular, due to competition in the deposit market, the deposit rates equal the central bank rate. Further, because laborers earn interest on their deposits, they accept lower wages. Thus we have:

Lemma 4.1. (Interest rates and wages with a central bank) When warehouses earn the central bank rate R^{CB} on deposits, in equilibrium, the deposit rates, lending rate, and wage are as follows:

$$R_0^D = R_1^D = R^L = R^{\text{CB}} \tag{26}$$

and

$$w = \big(R^{\text{CB}}\big)^{-2}. \tag{27}$$

The crucial takeaway from the result is that the warehouse pays a higher deposit rate when the central bank rate is higher. This means that the farmer's incentive constraint takes into account a higher return from depositing in a warehouse, but the same depreciation rate from private storage. Formally, with the central bank rate R^{CB}, the farmer's incentive constraint at Date 1 reads

$$R^{\text{CB}}\big(y - R^{\text{CB}}B\big) \ge (1-\delta)y, \tag{28}$$

or

$$B \le \frac{1}{R^{\text{CB}}}\left(1 - \frac{1-\delta}{R^{\text{CB}}}\right)y. \tag{29}$$

Observe that whenever farmers are not too highly levered—$B < y\big(2R^{\text{CB}}\big)^{-2}$—increasing R^{CB} loosens the incentive constraint. The reason is that it makes warehouse storage relatively more attractive at Date 1, inducing farmers to repay their debt rather than to divert capital.[29]

[28] We are considering a rather limited aspect of central bank monetary policy here, thereby ignoring things like the role of the central bank in setting the interest rate on interbank lending, as in Freixas et al. (2011), for example.

[29] The reason that increasing R^{CB} does not loosen the constraint when B is high, is that it also increases the lending rate between Date 0 and Date 1.

Proposition 11. (Monetary policy and liquidity creation) A tightening of monetary policy (an increase in R^{CB}) increases liquidity creation Λ as long as $\alpha + 2R^{CB}(1-\delta) > \left(R^{CB}\right)^2$ (otherwise it decreases liquidity creation).

This contrasts with the established idea that a lowering of the interest rates by the central bank stimulates bank lending.[30] In our model, high interest rates allow banks to lend more. This result complements Corollary 2, which says that liquidity creation is increasing in the depreciation rate δ. Both results say that the better warehouses are at storing grain relative to farmers, the more warehouses can lend.

5. Robustness

In this section, we show that the incentive mechanism is robust to two variations of the model, (i) in which warehouses store a second good, "gold," rather than grain and (ii) in which the farmer consumes at the interim date.

5.1. Grain or gold?

In the baseline model, the same good, grain, is both the consumption/production good and the store of value. Although the very early antecedents of banks were warehouses of grain, the direct predecessors of modern banks were warehouses of precious metals and valuables, as discussed earlier. That is, many early banks were places to safeguard durable goods that served as a store of value. As such, one might ask whether our analysis applies if warehouses do not have the superior ability to store (consumable) grain, but rather the superior ability to store a second good, such as gold, which is not directly used for consumption or production. In other words, if farmers have grain and warehouses have a superior ability to store gold, can warehouses still enforce repayment from farmers?

In this section, we argue that the answer is yes. To do this, we extend the model to include such a second good, "gold," and show that the warehouses' superior storage technology generates the incentive to repay loans, just as in the baseline model. To see why, suppose a farmer has produced grain and needs to save. Since grain is perishable, he uses his grain to buy gold. Now he considers whether to store this gold privately, in which case it depreciates, or store it in a warehouse, in which case he must repay his loan. This is the same tradeoff as in the baseline model, in which warehouses store grain, and it gives rise to the same incentive constraint.

Now consider the following three changes to the baseline model. (i) Grain is now fully perishable and cannot be stored, but is still the consumption/production good. (ii) There is a second good, gold, which is durable and can be stored. (iii) Gold depreciates at rate $\delta > 0$ unless it is warehoused. Here, we can interpret warehouses' preventing "depreciation" as capturing safeguarding against theft.

We let p denote the (exogenous) price of gold in terms of grain.[31] Thus, if a farmer has grain g at Date 1, he buys g/p units of gold. If he deposits his gold in a warehouse, the warehouse can seize the gold that the farmer owes it, whereas if he stores his gold privately, it depreciates at rate δ. The next proposition says the farmer's incentive constraint is the same as in the baseline model (Proposition 3).

Proposition 12. (Incentive constraint with two goods) If a farmer has grain g (gold g/p) and debt $R^L B$ at Date 1, he prefers to repay his debt and deposit the commodity rather than store it privately if and only if the incentive constraint in Eq. (IC) holds.

Like the incentive constraint in the baseline model, this result connects two economic functions of banks: to take deposits and to make loans. Introducing a durable store of value does not diminish warehouse banks' economic value, since this store of value must also be safeguarded against depreciation. Warehouses' advantage in doing this makes it incentive compatible for borrowers to repay, which facilitates bank lending and liquidity creation.

5.2. Consumption at Date 1

In the baseline model, farmers produce at Date 1 and consume only at Date 2. This gives them the need to save between Date 1 and Date 2, generating the demand to deposit in a warehouse to access its superior storage technology. In this section, we consider a different specification of farmers' preferences, in which farmers have log utility and consume at Date 1 and Date 2. We show that our mechanism is robust to the possibility of allowing farmers to consume at Date 1. The intuition for this is that with log utility the farmer has an incentive to smooth consumption across dates. Thus, he always has an incentive to save something for Date 2.

Here we denote consumption at Date t by c_t so that a farmer's total payoff is given by

$$U(c_1, c_2) = \log c_1 + \log c_2. \tag{30}$$

If a farmer has grain g at Date 1, it is incentive compatible for him to repay his debt if he prefers to deposit and repay than to divert, where now if he diverts he can either store privately, as before, or consume immediately. This generates an incentive constraint analogous to that in the baseline model—Eq. (IC)—except with a lower rate of depreciation, as summarized in the next proposition.

Proposition 13. (Borrowing constraint with consumption at Date 1) If farmers have logarithmic utility at Date 1 and Date 2, their borrowing constraint is given by

$$L \leq \frac{\sqrt{R_D} - \sqrt{1-\delta}}{\sqrt{R_D R_L}} g \tag{31}$$

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[30] See, for example, Keeton (1993). Mishkin (2010) provides a broad assessment of monetary policy, bank lending, and the role of the central bank.

[31] We take the price as exogenous here, for simplicity. However, it could also represent the equilibrium price in a model with another type of player; for example, a population of competitive "jewelers" could have value p for gold in terms of grain, leading to the endogenous price p.

or, in equilibrium,

$$L \le \left(1 - \sqrt{1-\delta}\right) g \approx \frac{\delta g}{2}, \tag{32}$$

where the approximation follows from the Taylor expansion.

6. Conclusion

In this paper, we have developed a new theory of banking that is tied to the origins of banks as commodity warehouses. The raison d'être for banks does not require asymmetric information, screening, monitoring, or even risk. Rather, we show that the institutions with the best storage (warehousing) technology have an advantage in enforcing contracts, and they are therefore not only the natural deposit-takers but are also the natural lenders—i.e. they are the natural banks. The development of interbank markets supports this enforcement mechanism in the presence of competing banks. Further, despite evolving from warehouses that store real goods, banks make loans by issuing "fake" warehouse receipts. Consequently, in our model, it is not only the case that deposits create loans, as in much of the existing literature, but also that loans create deposits.

Our theory has implications for contemporary bank regulation. It shows that higher levels of bank capital enhance bank liquidity creation. Moreover, we establish conditions under which tighter monetary policy induces more liquidity creation.

We hope that our framework is useful to explore other questions in future research. (i) Why do money-like assets have a price premium? (ii) What should the money supply be? (iii) How do firms decide between bank debt and market debt? To address each of these questions, we could extend our analysis (i) of warehousing a store of value ("gold") in Section 5.1, (ii) of monetary policy in Section 4.4, and (iii) of our baseline model to include risky projects and capital markets.

Appendix A. Proofs

Proof of Proposition 1. As discussed in the text preceding the statement of the proposition, in the first best all grain is invested in its first-best use at Date 0. This corresponds to $i_{\text{fb}} = e$, since the farmer's technology is the most productive. The production function requires $\ell_{\text{fb}} = \alpha i = \alpha e$ units of labor to be productive, and any more is unproductive. In summary, $i_{\text{fb}} = e$ and $\ell_{\text{fb}} = \alpha e$, as stated in the proposition. □

Proof of Proposition 2. First observe that the wage $w = 1$ since laborers' marginal cost of labor is one (Lemma 3.3 below). Now, substituting this into the budget constraint in Eq. (4) and solving the system with Eq. (5) gives the result immediately. □

Proof of Proposition 3. This result follows immediately from comparing a farmer's payoff from repaying his debt $R^L B$ and storing g in a warehouse at rate R^D_1, with his payoff from defaulting on his debt and storing g privately at rate $1-\delta$. Thus, he gets $R^D_1(g - R^L B)$ if he repays and deposits, and he gets $(1-\delta)g$ if he diverts and stores privately. The comparison of these expression gives the statement in the proposition. □

Proof of Proposition 4. A farmer has debt $R^L B$, which the warehouse trades in the interbank market. Without loss of generality, suppose that the warehouses trade bonds with face value one. Thus, the supply of the farmer's bonds is equal to its total outstanding debt $R^L B$. Denote the price of one bond by p.

We first show that if a farmer deposits his output in any warehouse, then his bonds trade at par. The key to the argument is that, because the warehouses that hold the farmers' grain can seize it at par, the debt cannot trade at a discount from par—if the price of debt is less than one, then the warehouses that hold it demand more than the total supply.

We assume that the incentive constraint in Proposition 3—that the farmer prefers to deposit and repay than to store privately—is satisfied. This is without loss of generality, since if it does not hold, the farmer will just store privately, i.e. he will not deposit in a warehouse at all, and there will be no trade in the interbank market. It follows from the IC that the farmer's total Date 1 deposits g exceed his total debt $R^L B$.

Lemma A.1. The price of the farmer's bonds in the interbank market is one, $p = 1$.

Proof. For this proof, we augment our notation slightly and denote the grain that the farmer has deposited in warehouse b by d^b and warehouse b's demand for the farmer's debt by x^b. The proof is by contradiction.

First suppose (in anticipation of a contradiction) that $p < 1$. Each warehouse b who holds the farmer's grain has demand for the farmer's debt equal to the deposits he has $x^b = d^b$. Thus, the total demand for the farmers' debt equals the total of his deposits g, $\int_b x^b db = \int_b d^b db = g$. This is greater than the total supply of the farmer's debt.[32] Thus, the market cannot clear. We conclude that it must be that $p \ge 1$.

Now suppose (in anticipation of a contradiction) that $p > 1$. All warehouses sell the farmer's debt, supplying $R^L B$, but no warehouse buys the farmer's debt, since the price is greater than any warehouse's private value (which is at most one). Thus, the market cannot clear. We conclude that it must be that $p \le 1$.

Since $p \ge 1$ and $p \le 1$, $p = 1$. □

We now conclude the analysis of the farmer's repayment. Trade in the interbank market results in warehouses that hold deposits buying all $R^L B$ units of the farmer's debt at price $p = 1$. These warehouses seize the total $R^L B$ units of the farmer's grain that they are owed—the farmer repays in full. Further, the warehouses that lend to the farmer either seize repayment from their deposits at par or sell his debt at par in the interbank market—the lending warehouses are repaid in full. □

Proof of Lemma 3.1. We show the result by contradiction. If $R^D_t \ne 1$ in equilibrium, deposit markets cannot clear.

[32] I.e. $g > R^L B$. This follows from the fact that the incentive constraint—Eq. (IC)—must be satisfied, as noted above.

First suppose (in anticipation of a contradiction) that $R_t^D < 1$ in equilibrium (for either $t \in \{0, 1\}$). Now set $s_t^b = D_t$ in the warehouse's problem in Section 3.4. The warehouse's objective function—Eq. (8)—goes to infinity as $D_t \to \infty$ without violating the constraints. The deposit markets therefore cannot clear if $R_t < 1$, a contradiction. We conclude that $R_t^D \geq 1$.

Now suppose (in anticipation of a contradiction) that $R_t^D > 1$ in equilibrium (for either $t \in \{0, 1\}$). Now set $s_t^b = D_t$ in the warehouse's problem. The warehouse's objective function goes to infinity as $D_t \to -\infty$ without violating the budget constraints. Thus, if $R_t^D > 1$, it must be that $D_t = 0$. However, since the depreciation rate $\delta > 0$, the demand from laborers and farmers to store grain is strictly positive for $R_t^D > 1 - \delta$. Thus, again, deposit markets cannot clear, a contradiction. We conclude that $R_t^D \leq 1$.

The two contradictions above taken together imply that $R_t^D = 1$ for $t \in \{0, 1\}$. □

Proof of Lemma 3.2. We show the result by contradiction. If $R^L \neq 1$ in equilibrium, loan markets cannot clear.

First suppose (in anticipation of a contradiction) that $R^L > 1$ in equilibrium. Now set $L = D_t$ in the warehouse's problem in Section 3.4. Given that $R_0^D = 1$ from Lemma 3.1 above, the warehouse's objective function—Eq. (8)—goes to infinity as $L \to \infty$ without violating the constraints. The deposit markets therefore cannot clear if $R^L > 0$, a contradiction. We conclude that $R^L \leq 1$.

Now suppose (in anticipation of a contradiction) that $R^L < 1$ in equilibrium. Now set $L = D_0$ in the warehouse's problem. Given that $R_0^D = 1$ from Lemma 3.1 above, the warehouse's objective function goes to infinity as $L \to -\infty$ without violating the budget constraints. Thus, if $R^L < 1$, it must be that $D_0 = 0$. However, since the depreciation rate $\delta > 0$, the demand from laborers and farmers to store grain is always strictly positive for $R_t^D > 1 - \delta$. Thus, again, deposit markets cannot clear, a contradiction. We conclude that $R^L \geq 1$.

The two contradictions above taken together imply that $R^L = 1$. □

Proof of Lemma 3.3. We show the result by contradiction. If $w \neq 1$ in equilibrium, labor markets cannot clear.

First suppose (in anticipation of a contradiction) that $w > 1$ in equilibrium. From Corollary 1,[33] $d_0^l = w\ell^l$ and $d_1^l = R_0^D d_0^l$ in the laborer's problem in Section 3.4. The constraints collapse, and the laborer's objective function—Eq. (10)—is $R_1^D R_0^D w\ell^l - \ell^l = (w-1)\ell^l$, having substituted $R_0^D = R_1^D = 1$ from Lemma 3.1 above. Since $w > 1$ by supposition, the objective function approaches infinity as $\ell^l \to \imath$ without violating the constraints. The labor market therefore cannot clear if $w > 1$, a contradiction. We conclude that $w \leq 1$.

Now suppose (in anticipation of a contradiction) that $w < 1$ in equilibrium. As above, the laborer's objective function is $(w-1)\ell^l$. Since $w < 1$ by supposition, the laborer sets $\ell^l = 0$. The farmer, however, always has a strictly positive demand for labor if $w < 1$—he produces nothing without labor and his productivity $A > 1 + 1/\alpha$ by Parameter Restriction 1. The labor market therefore cannot clear if $w < 1$, a contradiction. We conclude that $w \geq 1$.

The two contradictions above taken together imply that $w = 1$. □

Proof of Corollary 1. Given Lemma 3.1 above, the result is immediate from inspection of the farmer's problem and the laborer's problem in Section 3.4 given that $R_0^D = R_1^D = 1 > 1 - \delta$. □

Proof of Lemma 3.4. Before explaining the proof of the lemma, we state a preliminary result which says that, given the equilibrium prices established in Section 3.5, for any solution to the farmer's individual maximization problem, laborers' and warehouses' demands are such that markets clear.

Lemma A.2. (Warehouse and Laborer Preferences) Given the equilibrium prices, $R_0^D = R_1^D = R^L = w = 1$, warehouses are indifferent among all deposit and loan amounts and laborers are indifferent, among all labor amounts.

Proof. The result follows immediately from the proofs of Lemmas 3.1–3.3, which pin down the prices in the model by demonstrating that if prices do not make these players indifferent, markets cannot clear, contradicting that the economy is in equilibrium.

Now, the result follows from Lemma A.2 above and substituting in prices and demands from the preliminary results in Section 3.5. In short, since, given the equilibrium prices, laborers and warehouses are indifferent among allocations, they will take on the excess demand left by the farmers to clear the market. □

Proof of Proposition 5. We begin by rewriting the farmer's problem in Lemma 3.4 as

$$\text{maximize} \quad d_1^f \tag{A.1}$$

subject to

$$\delta\left(A\min\left\{\alpha i, \ell^f\right\} + d_0^f\right) \geq B, \tag{IC}$$

$$d_1^f + B = A\min\left\{\alpha i, \ell^f\right\} + d_0^f, \tag{BC$_1^f$}$$

$$d_0^f + i + \ell^f = e + B, \tag{BC$_0^f$}$$

and $i \geq 0$, $\ell^f \geq 0$, $B \geq 0$, $d_0^f \geq 0$, and $d_1^f \geq 0$.

Now observe that at the optimum, $\min\left\{\alpha i, \ell^f\right\} = \ell^f$ and $\ell^f = \alpha i$. Further, eliminate the d_1^f in the objective from the budget constraint. Now we can write the problem as

$$\text{maximize} \quad A\ell^f + d_0^f - B \tag{A.2}$$

subject to

$$\delta\left(A\ell^f + d_0^f\right) \geq B, \tag{IC}$$

$$d_0^f + i + \ell^f = e + B, \tag{BC$_0^f$}$$

$$\ell^f = \alpha i, \tag{A.3}$$

and $i \geq 0$, $\ell^f \geq 0$, $B \geq 0$, and $d_0^f \geq 0$.

[33] The proof of Corollary 1 is below; it does not depend on this result.

We see that the budget constraint and $\ell^f = \alpha i$ imply that

$$B = d_0^f + \frac{1+\alpha}{\alpha}\ell^f - e, \tag{A.4}$$

and thus the objective is

$$A\ell^f - \frac{1+\alpha}{\alpha}\ell^f + e = \frac{\alpha(A-1)-1}{\alpha}\ell^f + e. \tag{A.5}$$

This is increasing in ℓ^f by Parameter Restriction 1, so ℓ^f is maximal at the optimum. Thus, the incentive constraint binds, or

$$\delta\left(A\ell^f + d_0^f\right) = B = d_0^f + \frac{1+\alpha}{\alpha}\ell^f - e, \tag{A.6}$$

or

$$e - (1-\delta)d_0^f = \left(1 - \delta A + \frac{1}{\alpha}\right)\ell^f. \tag{A.7}$$

Since, by Parameter Restriction 2, $\delta A < 1$, setting $d_0^f = 0$ maximizes ℓ^f. Hence,

$$\ell^f = \frac{\alpha e}{1 + \alpha\left(1 - \delta A\right)}. \tag{A.8}$$

Combining this with the budget constraint and the equation $i = \ell^f/\alpha$ gives the expressions in the proposition. □

Proof of Proposition 6. We divide the proof of the proposition into three steps. In Step 1, we explain that a mechanism that implements the most severe feasible punishments can implement the (constrained) optimal outcome. In Step 2, we argue that the most severe punishments in our environment are the exclusion from warehousing. In Step 3, we show that our environment with Walrasian markets, in which warehouses can seize their deposits. implements these punishments.

Step 1. A mechanism can implement an outcome if the outcome is incentive compatible given the mechanism. Increasing the severity of punishments corresponds to loosening incentive constraints, which expands the set of implementable outcomes. Hence, increasing the severity of punishments expands the set of implementable outcomes.

Step 2. In our environment, punishments must be administered at Date 1 (at Date 2 agents consume, so we are effectively already in autarky, and at Date 0 it is too early to punish them for anything). At Date 1, there are only two technologies: private storage and warehouse storage. Thus, the only benefit the environment provides beyond autarky is access to warehousing. In other words, the worst possible punishment is exclusion from warehousing.

Step 3. The only limit to commitment in our environment comes from the nonpledgeability of farmers' output—the farmer is the only player who might not fulfill his promise. However, given the interbank market, anything the farmer deposits in the warehouse ultimately can be seized. Thus, the only way that a farmer can avoid repayment at Date 1 is by storing privately. This is equivalent to saying that if a farmer breaks his promise, he cannot store in a warehouse—he receives the autarky payoff. Thus, our model imposes the most severe feasible punishments on defecting players. As a result (from Step 1), our model implements the optimal incentive-feasible outcome. □

Proof of Proposition 7. The result that $\Lambda_{\text{nr}} = 1$ follows from the fact that farmers have the entire initial endowment. With no fake receipts, warehouses cannot lend because they have no initial grain endowment. (Laborers never lend, i.e. provide labor on credit, because they cannot enforce repayment from farmers.) Thus, there is no credit extended and no liquidity created: the equilibrium allocation equals the allocation with no credit in Proposition 2.

The second part of the result follows immediately from comparison of the equilibrium expression for $i + w\ell$ given in Proposition 5 and Parameter Restriction 2 . □

Proof of Corollary 2. The result is immediate from differentiation, as expressed in Eq. (17). □

Proof of Proposition 8. The expression given in the proposition is positive as long as $1 - \delta A > 0$. This holds by Parameter Restriction 2. The result follows immediately. □

Proof of Proposition 9. The proof comes from solving for the Date 1 deposits D_1 in the equilibrium in Proposition 5 and checking when the warehouse's incentive constraint—Eq. (20)—is violated. In the equilibrium in Proposition 5 we have that

$$D_1 = y + s_0^b \tag{A.9}$$

$$= A\alpha i + (e - i) \tag{A.10}$$

$$= \frac{\alpha\left[1 + (1-\delta)A\right]e}{1 + \alpha(1 - \delta A)}, \tag{A.11}$$

having substituted for i from the expression in Proposition 5. Thus, the warehouse's IC is violated for $e^b < \hat{e}^b$, where $\hat{e}^b$ solves

$$\frac{1-\delta}{\delta} = \frac{\hat{e}^b}{D_1} = \frac{\left[1 + \alpha(1 - \delta A)\right]\hat{e}^b}{\alpha\left[1 + (1-\delta)A\right]e}, \tag{A.12}$$

or

$$\hat{e}^b = \frac{1-\delta}{\delta}\frac{\alpha\left[1 + (1-\delta)A\right]}{1 + \alpha(1 - \delta A)}e. \tag{A.13}$$

□

Proof of Proposition 10. This result follows from solving for the equilibrium with the warehouse's incentive constraint binding. We proceed assuming that lending L is positive. If it is negative, the formulae do not apply and $L = 0$.

Begin with the warehouses' binding incentive constraint, which gives a formula for D_1, the total grain deposited at Date 1,

$$D_1 = \frac{\delta}{1-\delta}e^b. \tag{A.14}$$

The Date 1 deposit market clearing condition implies that the total amount of deposits equals the total amount of grain at Date 1. This is the sum of the farmer's output y and the grain stored in the warehouse at Date 0, s_0^b,

$$D_1 = y + s_0^b \tag{A.15}$$

$$= A\alpha i + e - i. \tag{A.16}$$

Combining this with the warehouses' incentive constraint gives

$$i = \frac{1}{\alpha A - 1}\left(\frac{\delta}{1-\delta}e^b - e\right). \tag{A.17}$$

(Note that $\alpha A - 1 > 0$ by Parameter Restriction 1.) Now, since the farmers' technology is Leontief, $\ell = \alpha i$. This allows us to write the expression for the liquidity multiplier Λ:

$$\Lambda = \frac{i + w\ell}{e} \tag{A.18}$$

$$= \frac{1+\alpha}{\alpha A - 1}\left(\frac{\delta}{1-\delta}\frac{e^b}{e} - 1\right). \tag{A.19}$$

This expression applies when it is greater than one (and e^b is below the threshold in Proposition 9) or

$$\frac{1+\alpha}{\alpha A - 1}\left(\frac{\delta}{1-\delta}\frac{e^b}{e} - 1\right) \geq 1. \tag{A.20}$$

Which can be rewritten as

$$e^b \geq \frac{\alpha(1-\delta)(1+A)e}{(1+\alpha)\delta} \equiv \hat{e}^b. \tag{A.21}$$

Otherwise, no liquidity is created and the liquidity multiplier is one. □

Proof of Lemma 4.1. The proofs that $R_0^D = R_1^D = R^L = R^{CB}$ are all identical to the proofs of the analogous results in Section 3.5 with the warehouses' return on storage (which is one in the baseline model) replaced with the central bank rate R^{CB}. The result is simply that warehouses lend and borrow at their cost of storage, which is a result of warehouses being competitive.

The result that $w = \left(R^{CB}\right)^{-2}$ is also nearly the same as the proof of the analogous result (Lemma 3.3) in Section 3.5. The modification is that the laborer's objective function—Eq. (10)—reduces to $c^l = \left(R^{CB}\right)^2 w\ell - \ell$, since the laborer invests its income in the warehouse for two periods at gross rate R^{CB}. In order for the laborer not to supply infinite (positive or negative) labor ℓ, it must be that $w = \left(R^{CB}\right)^{-2}$. □

Proof of Proposition 11. Solving for the equilibrium again reduces to solving the farmer's problem with binding incentive and budget constraints. With the prices given in Lemma 4.1 these equations are

$$R^{CB}\left(y - R^{CB}\right) = (1-\delta)y, \tag{A.22}$$

and

$$i + \left(R^{CB}\right)^{-2}\ell = e + B, \tag{A.23}$$

where $y = A\min\{\alpha i, \ell\}$ and, in equilibrium, $i = \alpha\ell$. From the budget constraint we find that

$$\ell = \frac{\alpha\left(R^{CB}\right)^2(e+B)}{\alpha + \left(R^{CB}\right)^2} \tag{A.24}$$

and, combining the above with the incentive constraint,

$$B = \frac{\alpha A\left(R^{CB} - 1 + \delta\right)e}{\alpha + \left(R^{CB}\right)^2 - \alpha A\left(R^{CB} - 1 + \delta\right)}. \tag{A.25}$$

This gives the following equilibrium allocation:

$$\ell = \frac{\alpha\left(R^{CB}\right)^2 e}{\alpha + \left(R^{CB}\right)^2 - \alpha A\left(R^{CB} - 1 + \delta\right)}, \tag{A.26}$$

$$i = \frac{\left(R^{CB}\right)^2 e}{\alpha + \left(R^{CB}\right)^2 - \alpha A\left(R^{CB} - 1 + \delta\right)}. \tag{A.27}$$

We use the allocation to write down the liquidity multiplier Λ as

$$\Lambda = \frac{i + w\ell}{e} \tag{A.28}$$

$$= \frac{\alpha + \left(R^{CB}\right)^2}{\alpha + \left(R^{CB}\right)^2 - \alpha A\left(R^{CB} - 1 + \delta\right)}. \tag{A.29}$$

We now compute the derivative of Λ with respect to R^{CB} to show when increasing R^{CB} increases Λ:

$$\frac{\partial\Lambda}{\partial R^{CB}} = \frac{2R^{CB}\left[\alpha + \left(R^{CB}\right)^2 - \alpha A\left(R^{CB} - 1 + \delta\right)\right] - \left(\left(R^{CB}\right)^2 + \alpha\right)\left(2R^{CB} - \alpha A\right)}{\left[\alpha + \left(R^{CB}\right)^2 - \alpha A\left(R^{CB} - 1 + \delta\right)\right]^2}$$

$$= \frac{\alpha A\left[\alpha + 2(1-\delta)R^{CB} - \left(R^{CB}\right)^2\right]}{\left[\alpha + \left(R^{CB}\right)^2 - \alpha A\left(R^{CB} - 1 + \delta\right)\right]^2}.$$

This is positive exactly when $\alpha + 2R^{CB}(1-\delta) > \left(R^{CB}\right)^2$ as stated in the proposition. □

Proof of Proposition 12. Analogously to the proof of the incentive constraint in the baseline model (Proposition 3), the result follows immediately from comparing a farmer's payoff from repaying his debt $R^L B$ and storing gold g/p in a warehouse at rate R_1^D with his payoff from defaulting on his debt and storing g/p privately at rate $1-\delta$. The twist is that now we must take into account the prices of gold.

First consider the farmer's payoff if he deposits in the warehouse. Since grain is the numeraire, the value (in grain) of his deposits is $g - R^L B$ at Date 1 and $R_1^D(g - R^L B)$ at Date 2. Next consider the farmer's payoff if he stores his gold g/p privately. He has $(1-\delta)g/p$ units of gold at Date 2. This is worth $p \times (1-\delta)g/p = (1-\delta)g$ in grain. Thus, he prefers to repay and deposit than to store privately if and only if

$$R_1^D(g - R^L B) \geq (1-\delta)g. \tag{A.30}$$

This is just the incentive constraint in Eq. (IC), as desired. □

Proof of Proposition 13. If a farmer has grain g at Date 1, it is incentive compatible for him to repay his debt if he prefers to deposit and repay than to divert, where now if he diverts he may either store privately, as before, or consume immediately. His payoff if he does not divert is

$$U_{\text{deposit}} = \max\left\{ u(c_1) + u(c_2) \;\middle|\; c_2 = R_D(g - c_1 - R_L L) \right\}. \tag{A.31}$$

Solving the program with the first-order approach gives

$$U_{\text{deposit}} = \log\left(\frac{g - R_L L}{2}\right) + \log\left(\frac{R_D(g - R_L L)}{2}\right). \tag{A.32}$$

Likewise, the payoff if the farmer does divert is

$$U_{\text{divert}} = \max\left\{ u(c_1) + u(c_2) \;\middle|\; c_2 = (1-\delta)(g - c_1) \right\}. \tag{A.33}$$

Solving this program with the first-order approach gives

$$U_{\text{divert}} = \log\left(\frac{g}{2}\right) + \log\left(\frac{(1-\delta)g}{2}\right). \tag{A.34}$$

Now we can write the farmer's incentive constraint with log utility. He prefers to deposit than to divert if $U_{\text{deposit}} > U_{\text{divert}}$ in Eqs. (A.31) and (A.33) above. Thus the borrowing constraint is given by

$$L \le \frac{\sqrt{R_D} - \sqrt{1-\delta}}{\sqrt{R_D} R_L} g, \tag{A.35}$$

as stated in the proposition. Above, we have used the fact that $\log x + \log y = \log xy$ and simplified. If we substitute $R_D = R_L = 1$ and use the Taylor approximation, we can express the borrowing constraint as follows:

$$L \le \left(1 - \sqrt{1-\delta}\right) g \approx \frac{\delta g}{2}. \tag{A.36}$$

This is exactly the incentive constraint in the model with linear utility and consumption only at Date 2 and rate of depreciation $\delta/2$. Thus, we conclude that our basic mechanism is not affected by consumption at the interim date, although it can attenuate the importance of savings. Specifically, it corresponds to lowering the rate of depreciation. □

Appendix B. Table of notations.

Indices	
f	Farmer index
l	Laborer index
b	Warehouse (bank) index
$t \in \{0, 1, 2\}$	Time index
Prices	
R_t^D	Deposit rate at Date t
R^L	Lending rate at Date 0
w	Wages at Date 0
Demand and supply	
i	Grain farmers invest at Date 0
ℓ^f	Labor farmers demand at Date 0
ℓ^l	Labor laborers supply at Date 0
s_t^j	Grain stored by player j at Date t
d_t^f	Grain deposited in warehouses by farmers at Date t
d_t^l	Grain deposited in warehouses by laborers at Date t
B	Loans demanded by farmers at Date 0
L	Loans supplied by warehouses at Date 0
D_t	Overall deposits in warehouse at Date t
Production and consumption	
y	Farmers' output at Date 1
c^j	Consumption of player j at Date 2
Parameters	
δ	Depreciation rate with private storage
A	Productivity
α	Ratio of labor to grain in farmers' production
e	Farmers' (Date 0) endowment
e^b	Warehouses' (Date 1) endowment (extension in Section 4.3)
$\hat{e}^b$, $\hat{\hat{e}}^b$	Thresholds of warehouse capital determining lending behavior (extension in Section 4.3)
p_t	Price of gold at Date t (extension in Section 5.1)
Other variables	
g_1^f	Farmer's grain holding at Date 1
Λ	Liquidity multiplier
R^{CB}	Central bank rate (extension in Section 4.4)

(*continued on next page*)

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